## S.m.11. NIRMALA ANTHERJANAM, N.–Studies on Korteweg-De Vries Equations–1988–Dr. M. Jathavedan

The korteweg-de Vries (KdV) equation is one the most widely studied evolution equations in non-linear waves, which was derived in the last century by Korteweg and de Vries as a model equation for nonlinear dispersive long waves (1895). The most outstanding property of this equation is that is admits solitary waves, first reportedo by J.S. Russell (1845) as solutions. The interest in the study of KdV equation was revived after a numerical expriment by Sabusky and Kruskal in 1965. They found that two solitary waves after interaction emerge unchanged but for a phase shift. Since then KdV equation has found application in different fields like gravity waves, plasma waves and waves in lattices.

The thesis deals with studies on KdV equations. Chapter I is introductory. It contains a brief account of nonlinear hyperbolic waves, dispersive waves and the KdV equation in the context of water waves.

R.S. Johnson has found (1973) that the propagation of waves on shallow water, when the depth changes abruptly forming a shelf, is given by a KdV equation, the coefficients being functions of the far field space coordinate. In Chapter II we study the interaction of waves given by this equation. The derivative expansion method is applied. It is seen that three waves interaction is possible and there is exchange of energy between different wave numbers. The total energy is not conserved during interaction.

Johnson's equation is only a special case of the general form of KdV equation. There are many other KdV type equations arising in different contexts. In chapter III we give an account of these. We introduce a KdV type equation with variable coefficients. In the remaining chapters we study this equation.

The test of integrability is an important branch of study of nonlinear evolution equations. The Painleve Property (PP) is known to be related to the integrability of ordinary differential equations. In recent years the Painleve analysis has been extended to the study of partial differential equations also. In Chapter-IV we study the integrability of the above KdV equation using the method proposed by J. Weiss, M. Tabor and G. Carnevali (1983). The auto-Backlund transformation and Lax pairs are obtained by this method. Lax pair criterion enables to find the cases in which the equation is integrable.

In many cases when a differential equation cannot be integrated we can study the properties by qualitative methods. Similarity analysis is one such qualitative method which can give valuable informations regarding the solutions without solving the equation completely. Chapter-V is devoted to a similarity analysis of the KdV equation. The Ablowitz-Ramani-Segur (ARS) conjecture is used to identify the integrability of the equation. It is found that in some special cases the equation may be integrable. The exact solution in a particular case is obtained.

The Weiss et al. analysis is not a fully satisfactory method for testing the integrability and it is known that the method may lead to incorrect conclusions, atleast in some cases. But when a non-autonomous KdV system is integrable it can be transformed into an autonomous integrable KdV system by a suitable transformation. In the last chapter it is shown that the two cases in which the equation is integrable. It can be transformed into the plane KdV or cylindrical KdV equations. Since the integrability of the equation in these two cases is already known it follows that the method of Weiss et al. leads to correct conclusions in the case of this equation.