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Suppression of chaos through reverse period doubling in coupled directly modulated semiconductor lasers

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Abstract

The effect of coupling on two high frequency modulated semiconductor lasers is numerically studied. The phase diagrams and bifurcation diagrams are drawn. As the coupling constant is increased the system goes from chaotic to periodic behavior through a reverse period doubling sequence. The Lyapunov exponent is calculated to characterize chaotic and periodic regions. © 1999 Elsevier Science B.V.

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1. Introduction

Coupled nonlinear dynamical systems have been widely studied in the last few years. They exhibit a wide range of interesting phenomena and have found application in a variety of different fields. The dynamics of coupled systems like neural networks [1], chemical oscillators [2], coupled maps [3], Josephson junction [4], etc. have been studied. Coupled laser systems have also been studied [5]. However, many of the earlier studies on coupled laser systems were concerned with the spatial properties of the output radiation rather than their intrinsic dynamical properties. Recently Liu et al. [6] have studied the effect of coupling of two chaotic CO_2 lasers with saturable absorbers and Thornburg et al. [7] have studied the chaotic behavior of two coupled single mode Nd-YAG lasers. In this paper we report the results of our numerical studies on two coupled chaotic semiconductor lasers with high frequency current modulation. Semiconductor lasers with high frequency current modulations are important for large capacity information transmission, for ultrafast optical processing and for high speed pulse generation. Such lasers

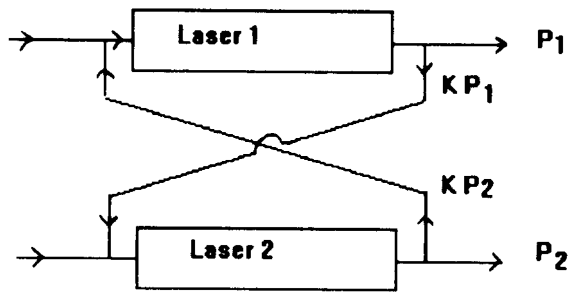

Fig. 1. Schematic diagram for coupled lasers.

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have been shown to exhibit chaotic behavior for certain parameter values, both experimentally [8] and theoretically [9,10].

The influence of the nonlinear gain suppression factor [11], spontaneous emission factor [12], Auger combination factor [13], and the effect of noise on such lasers [14] have been studied. Agrawal [11] has shown that the nonlinear gain suppression factor can eliminate the period doubling route to chaos in current modulated semiconductor lasers. However, the chaotic behavior is possible when the gain suppression factor is low.

2. The model

The numerical studies on current modulated semiconductor lasers were based on a set of rate equations for the photon density (S) and carrier density (n). The current modulation provides the third degree of freedom necessary for the generation of chaos. Considering the nonlinear gain suppression factor and the effect of noise, the rate equations can be written as [14]

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{I}{eV} - \frac{n}{\tau_e} - A(n - n_0)S + \frac{F_n(t)}{V}, \qquad (1)$$

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \Gamma A(n-n_0)(1-\varepsilon_{NL}S)S - \frac{S}{\tau_p} + \frac{\Gamma\beta n}{\tau_e} + \frac{F_s(t)}{V}, \qquad (2)$$

where

$$I = I_b + I_m \sin(2\pi f_m t) \qquad (3)$$

is the driving current, e is the charge of the electron, V is the active volume, τ_e and τ_p are the electron and photon life times respectively, A is the gain constant, n_0 is the carrier density required for transparency, Γ is the confinement factor, β is the spontaneous emission factor, ε_{NL} is the nonlinear gain reduction occurring with increase in S, I_b is the bias current, I_m is the modulation current and f_m is the modulation frequency. $F_s(t)$ and $F_n(t)$ are the Langevin noise sources with zero mean that arise respectively from the spontaneous emission and from the discrete nature of carrier generation and recombination.

The photon density and carrier density can further be normalized for numerical purposes by defining $N = n/n_{th}$ and $P = S/S_0$ with $S = \Gamma(\tau_p/\tau_e)\, n_{th}$ where n_{th} is the threshold carrier density. $n_{th} = \tau_e I_{th}/eV$, I_{th} is the threshold current. Correspondingly, neglecting the effect of noise, the rate equations become [11]

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{1}{\tau_e}\left(\frac{I}{I_{th}} - N - \left(\frac{N-\delta}{1-\delta}\right)P\right), \qquad (4)$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{\tau_p}\left(\frac{N-\delta}{1-\delta}(1-\epsilon P)P - P + \beta N\right), \qquad (5)$$

where $\delta = n_0/n_{th}$ and $\epsilon = \varepsilon_{NL}S_0$ are dimensionless parameters. For certain parameter values the output power shows chaotic behavior [11]. In this paper we have considered the coupling of two such identical chaotic lasers coupled in such a way that a current in milliamperes proportional to the output power of each laser is electronically fed to the other as shown schematically in Fig. 1. The corresponding rate equations can be written as

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = \frac{1}{\tau_e}\left(\frac{I_1}{I_{th}} - N_1 - \left(\frac{N_1-\delta}{1-\delta}\right)P_1\right), \qquad (6)$$

$$\frac{\mathrm{d}P_1}{\mathrm{d}t} = \frac{1}{\tau_p}\left(\frac{N_1-\delta}{1-\delta}(1-\epsilon P_1)P_1 - P_1 + \beta N_1\right), \qquad (7)$$

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = \frac{1}{\tau_e}\left(\frac{I_2}{I_{th}} - N_2 - \left(\frac{N_2-\delta}{1-\delta}\right)P_2\right), \qquad (8)$$

$$\frac{\mathrm{d}P_2}{\mathrm{d}t} = \frac{1}{\tau_p}\left(\frac{N_2-\delta}{1-\delta}(1-\epsilon P_2)P_2 - P_2 + \beta N_2\right), \qquad (9)$$

$$I_1 = I_b + I_m \sin(2\pi f_m t) + KP_2, \qquad (10)$$

$$I_2 = I_b + I_m \sin(2\pi f_m t) + KP_1. \qquad (11)$$

Here K is the proportionality factor and the subscripts 1 and 2 represent the first and second lasers respectively.

3. Numerical results

Eqs. (6)-(11) were studied numerically for various values of the coupling constant K and the results are presented. The numerical integrations were done using a Runge-Kutta fourth order method. The values of parameters used for numerical calculations are given in Table 1. In Fig. 2 we present the phase portrait of laser 1 (power versus carrier density) for various values of the coupling constant K. When K is small ($K = 2$) the phase portrait shows chaotic behavior. As K is increased the nature of the orbit changes, at

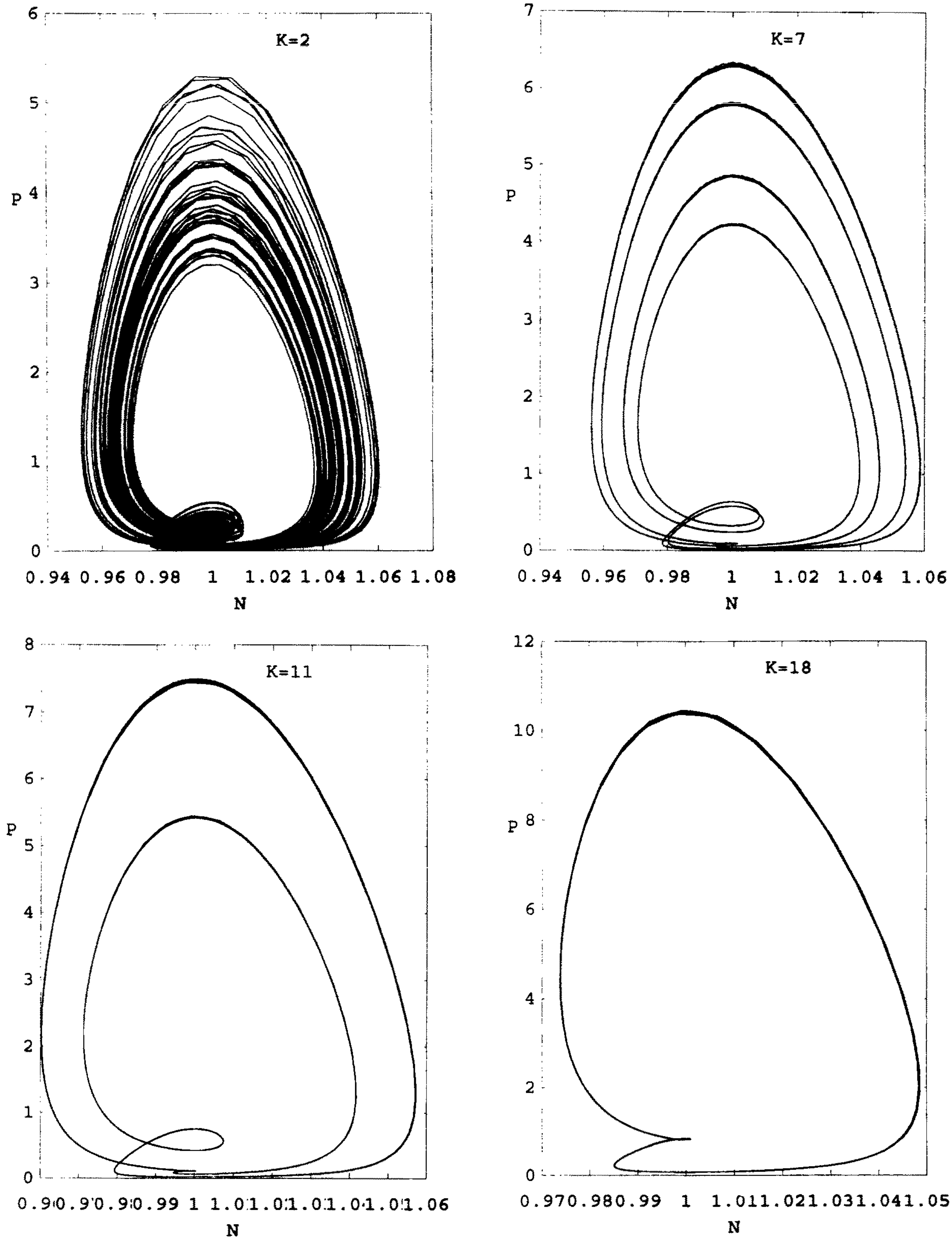

Fig. 2. Phase portrait (power versus carrier density) for laser 1 when the coupling constant $K = 2$, $K = 7$, $K = 11$ and $K = 18$.

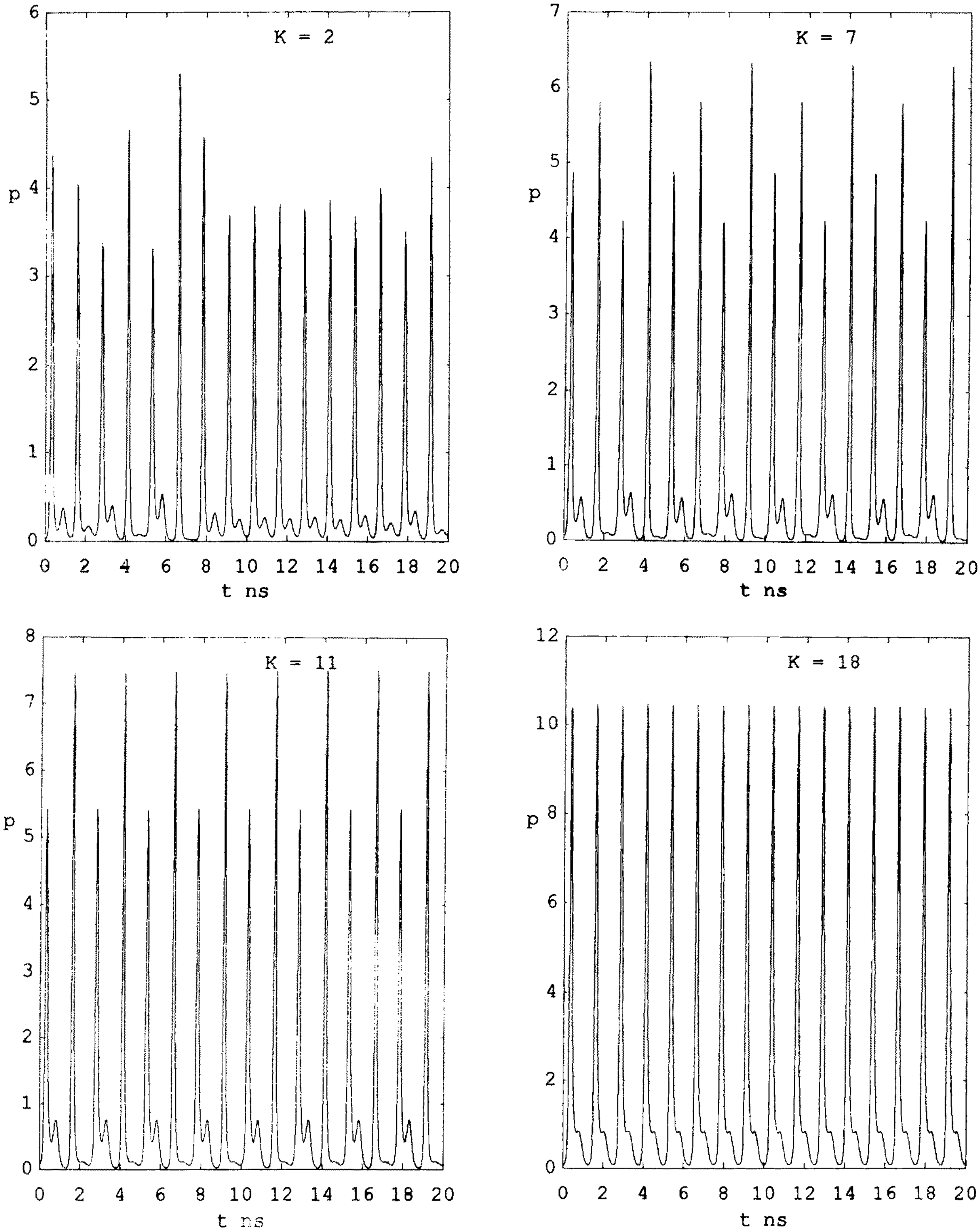

Fig. 3. Pulse train (power versus time) for laser 1 when the coupling constant $K = 2$ (chaotic), $K = 7$ (period 4), $K = 11$ (period 2) and $K = 18$ (period 1).

Table 1
Parameter values used for numerical calculations

Parameter	Value
τ_e, electron life time	3 ns
τ_p, photon life time	6 ps
β, spontaneous emission factor	5×10^{-5}
$\delta = n_0/n_{th}$	0.692
ϵ, nonlinear gain suppression factor	0.0001
f_m, modulation frequency	8 Mhz
I_{th}, threshold current	26 mA
I_b, bias current	$1.5 I_{th}$
I_m, modulation current	$0.3 I_{th}$

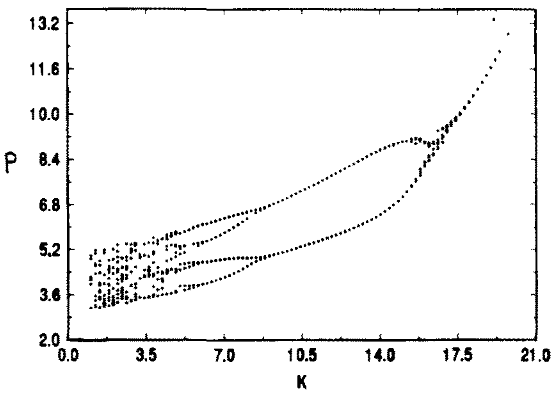

Fig. 4. Bifurcation diagram for the laser.

$K = 7$ we see a four-cycle. As K is increased further, the phase diagram exhibits a reverse period doubling bifurcation. For $K = 11$ the phase diagram shows a two-cycle and for $K = 18$ it shows a one-cycle. This behavior is also seen from Fig. 3 where the power P is plotted against time for the same values of K. The power variations are chaotic for $K = 2$ while it is periodic, with period four for $K = 7$, with period two for $K = 11$ and with period one for $K = 18$. The second laser also showed similar behavior.

Fig. 4 shows the bifurcation diagram for the laser. It is seen from the bifurcation diagram that as the coupling constant K is increased the system goes from chaotic to steady state (one cycle) through a sequence of reverse period doubling. This transition to periodic behavior is also evident from the values of the Lyapunov characteristic exponent (LCE) calculated from numerically obtained output power time series. The Wolf et al. algorithm [15] is used for the calculation of the LCE. The LCE calculated for different K values are given in Table 2. Again it is seen that for $K = 2$ the LCE is positive showing chaotic behavior while it is negative for higher values of K, showing regular behavior.

Table 2
Lyapunov exponents calculated from numerically obtained output power time series with the picosecond as the unit of time

K	Lyapunov exponent
2	$+1.928 \times 10^{-4}$
7	-1.864×10^{-4}
11	-3.662×10^{-4}
18	-3.682×10^{-4}

4. Conclusion

Our numerical studies show that the coupling of two lasers as described here has the effect of suppressing chaos of the lasers. By increasing the coupling constant K the system is made to undergo a cascade of inverse period doubling bifurcations. By suitably choosing the K values one can bring the output to any desired cycle.

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