Studies on Spatial and Temporal solitons with varying dispersion, diffraction and nonlinearity

Thesis submitted to

Cochin University of Science and Technology

in partial fulfillment of the requirements
for the award of the degree of

Doctor Of Philosophy

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September 2007

CERTIFICATE

Certified that the work presented in this thesis is a bonafide work done by Mrs.P.A. Subha, under my guidance in the Department of Physics, Cochin University of Science and Technology and that this work has not been included in any other thesis submitted previously for the award of any degree.

Kochi
September, 2007

Dr. V. C. Kuriakose
(Supervising Guide)

DECLARATION

I hereby declare that the work presented in this thesis is based on the original work done by me under the guidance of Dr. V. C. Kuriakose, Professor (Rtd.), Department of Physics, Cochin University of Science and Technology and has not been included in any other thesis submitted previously for the award of any degree.

Kochi
September, 2007

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Acknowledgements

Almost all real systems are nonlinear. For a nonlinear system the superposition principle breaks down; the system's response is not proportional to the stimulus it receives; the whole is more than the sum of its parts!

I was introduced to the exciting world of nonlinear physics by Prof.V.C. Kuriakose whose extensive knowledge and expertise in the field guided me to carry out this research work in the Department of Physics, Cochin University of Science and Technology. It's my pleasure to express enormous gratitude and record deepest sense of appreciation to him for his deep involvement, continuous encouragement and also for the very meaningful and stimulating discussions. His keen insight and creative ideas provided the platform and precise guidelines for my efforts to understand the complexities of nonlinear optics.

I am extremely grateful to Prof.T. Ramesh Babu, Head of the Department of Physics, Cochin University of Science and Technology for the fruitful and encouraging discussions and also for providing me the necessary facilities to accomplish this research. I remain deeply indebted to Prof. M. Sabir, Department of Physics for the excellent motivation and support rendered to me throughout the course of this work.

I really enjoyed the company of my co-researchers in the theoretical division, C.P. Jisha, Chithra R. Nayak, R. Radhakrishnan, R. Sini, O.K Vinayaraj, M .Vivek, Nijo, Prajitha, Priyesh and all other research scholars who sustained me with encouragement and assisted me a lot during the course of this work. My special thanks also go to all the faculty members, non-teaching staff of the department who have extended a helping hand for the fulfillment of my task.

I express my deep sense of gratitude to Prof. K. Porsezian and Mr. M.C. Padmarajan for their inspiring and valuable suggestions through discussions on the topic. I wish to acknowledge Mr.V.Kabeer and Mr.Satheesh for helping me in the numerical simulations.

Farook College, Kozhikode ever remains in the forefront for academic enrichment of its faculty in the field of research. The Management of the College, Prof.A. Kuttialikutty, Principal and Dr.P.M. Mubarak Pasha, former Principal deserve special mention for granting permission to undergo the course on Faculty Improvement Programme. I would also like to thank my colleagues in the Department of Physics for their keen interest and encouragement in my research work.

My special thanks are also recorded hereby to the University Grants Commission, New Delhi and also to the Government of Kerala for granting teacher fellowship and sanctioning deputation without which this work would have been impossible.

And of course, the progress of a task undertaken is greatly influenced by the love and support one enjoys in the company one belongs to. Let me take this opportunity to express my gratitude and appreciation to all my friends and relatives who supported and encouraged me in various ways during the course of this work.

P.A. Subha

Contents

Preface

The term soliton refers to special kinds of waves that can propagate undistorted over long distances and remain unaffected after collision with each other. In the twentieth century, the soliton became one of the most fruitful concepts of nonlinear physics and a key to understanding how nonlinearity acts in nature. The field of optical solitons have grown enormously due to its potential applications in communication, all optical switching, logic gates etc. In nonlinear optics, optical solitons are classified as temporal and spatial depending on whether the confinement of light occurs in time or space respectively during wave propagation. Both types of optical solitons have been the focus of extensive research.

This thesis presents a study of optical soliton propagation in inhomogeneous single mode and coupled fibres, and the stabilization of multidimensional spatial solitons. **Chapter 1** is a general introduction to temporal and spatial optical solitons. It is then clarified what has been explored in the area covered by the thesis.

The pulse propagation in an inhomogeneous optic fibre medium with gain(loss) and frequency chirping has been analyzed within the integrable limit in **Chapter 2**. The two soliton solution for the nonlinear Schrodinger equation (NLSE) has been constructed using Backlund transformation technique and a recursive method.

Chapter 3 deals with the study of soliton propagation in an inhomogeneous optic fibre system with varying dispersion. The NLSE representing the dynamics of this inhomogeneous optic fibre system is not integrable and cannot be solved analytically in general. The variational analysis, a semi analytical approach, based on Lagrangian formalism of classical mechanics has been used to study the system. The inhomogeneous optic fibre system has been studied numerically using split-step Fourier method. **Chapter 4** presents the propagation of optical solitons in two mode nonlinear directional coupler (NLDC) with inhomogeneous dispersion and nonlinearity. The soliton dynamics in NLDC is governed by coupled nonlinear Schrodinger equations. The switching dynamics in NLDC has been studied using variational analysis and numerical methods. An important application of fibre couplers consists of using them for all optical switching.

Spatial and spatiotemporal solitons have been suggested to be ideal candidates for a number of potential practical applications including all-optical soliton steering and switching. A challenging issue is the search for physically relevant models in which stable higher dimensional spatial solitons exist. In next three chapters, we have formulated and studied certain models in which stable higher dimensional spatial solitons exist.

The NLSE which governs the dynamics of two dimensional spatial solitons in Kerr media with periodically varying diffraction and nonlinearity has been analyzed in **Chapter 5** using variational approach and numerical studies. Analytical expressions for soliton parameters have been derived using variational analysis. Variational equations and partial differential equation have been simulated numerically.

We then show that the existence of stable three dimensional spatiotemporal solitons in bulk Kerr media with periodically varying dispersion, diffraction and nonlinearity in **Chapter 6** by means of variational approximation and Kapitsa averaging method.

In **Chapter 7** the stabilization of two dimensional spatial solitons and three dimensional spatiotemporal solitons in cubic-quintic media have been explored. A diffraction managed model in cubic-quintic media has been formulated and studied. A dispersion managed model with diffraction management has been proposed for the stabilization of three dimensional spatiotemporal soliton in bulk cubic-quintic media . The cubic-quintic nonlinear Schrodinger equation with periodically varying dispersion and diffraction has been studied analytically and numerically. In **Chapter 8**, main results and conclusion of this thesis are presented. Future plans for the extension of the work are also mentioned.

List of papers published/presented/communicated:

1. Multi soliton propagation in inhomogeneous optic fibre medium with varying dispersion (J. Nonlin. Opt. and Mat. **15**, 415, 2006)

2. Stable diffraction managed spatial soliton in bulk cubic-quintic media (J. Modern opt. DOI: 10.1080/09500340701197515, Published on: 17 April 2007).

3. Nonlinearity management and diffraction management for the Stabilization of two dimensional spatial solitons (Pramana J. Phys. **69**, 229, 2007)

4. Three dimensional light bullets in cubic-quintic media stabilized by periodic variation of diffraction and dispersion (communicated to European Physical Journal D)

5. Soliton switching in nonlinear directional coupler with inhomogeneous dispersion and nonlinearity (to be communicated)

6. Stable spatiotemporal solitons in Kerr media(to be communicated)

7. Diffraction managed two dimensional spatial solitons in cubic-quintic media

   (Nonlinear systems and dynamics edited by M .Lakshmanan and R.Sahadevan Allied publishers, 2006)

8. Nonlinearity management and diffraction management for the stabilization of two dimensional spatial solitons in Kerr media (Eighth International Conference on Optoelectronics, Fiber Optics and Photonics. Photonics 2006: Hyderabad, India).

9. Soliton switching in nonlinear directional coupler with varying dispersion and nonlinearity (Eighth International Conference on Optoelectronics, Fiber Optics and Photonics. Photonics 2006: Hyderabad, India)

Chapter 1

Introduction and Thesis outline

1.1 Basic concepts and terminology

The term soliton was coined by Zabusky and Kruskal [1] in 1965 to reflect the particle-like nature of solitary waves that remained intact even after mutual collisions. Soliton is a non-linear wave which has the following properties. a) a localised wave propagates without change of its shape and velocity. b) localized waves are stable against mutual collision and retain their characteristics. The first observation of the solitary wave was made in 1834 by a Scottish Engineer John Scott Russel and the word Solitary wave was coined by Scott Russel himself [2]. The theoretical confirmation for this phenomenon was given by two Dutch Physicists, Korteweg and de Vries, in the form of a nonlinear partial differential equation now known as KdV equation [3]. Zabusky and Kruskal solved the KdV equation numerically and observed that waves with sharp peaks emerge from a smooth initial waveform. Those pulse-waves move almost independently with constant speeds and pass through each other after collisions and thus the soliton was discovered.

Solitons are stable inspite of mutual interactions because the model equation (KdV) has an infinite number of conserved quantities. Dynamical properties of the system are severely restricted by the existence of an infinite number of conservation laws. The conserved quantities guarantee the time independence of parameters which characterize the solitons, and therefore the solitons are stable. Corresponding to an infinite number of conserved quantities(the field variable has infinite degrees of freedom), arbitrary number of solitons may co-exist. Fundamental properties of solitons are investigated by the inverse scattering transform method (IST) [4]. Zakharov and Shabat [5] solved the nonlinear Schrodinger equation (NLSE) by extending the IST. NLSE is one of the universal nonlinear wave equations widely applied to the study of problems in plasma physics, condensed matter physics, nonlinear fibre optics etc. Great success has been attained on the elaboration of novel approaches to soliton theory when IST has been developed. When the scattering data space is regarded as the extension of the momentum space, the inverse scattering method is considered as the extension of the Fourier transformation into nonlinear problems.

In the twentieth century, the word soliton became one of the most fruitful concepts of non-linear physics and a key to understanding how nonlinearity acts in nature. Solitons have become widespread interdisciplinary field of theoretical and experimental studies, in branches ranging from fluid dynamics, oceanography, plasma physics, solid state physics, particle physics, electrodynamics, nonlinear optics, etc. Yet over the past decade, the forefront of soliton research has shifted to nonlinear optics.

1.2 Optical solitons

A general property of electromagnetic wavepackets is that they tend to spread out as they evolve. A fundamental cause for this is that distinct frequency components which are superposed to create the wavepacket, propagate with different velocities and/or in different directions. An example is the transverse spreading of a laser beam due to diffraction. Similarly, light pulses spread in time as they propagate in a material medium, as due to the group-velocity dispersion (GVD), each Fourier component of the pulse has a different velocity. These examples pertain to linear propagation of beams or pulses. Nonlinear effects generally accelerate the disintegration of a wavepacket. However, under special conditions, nonlinearity may compensate the linear effects. The resulting balanced and localized pulse or beam of light, that propagates without decay, is generally known as a soliton or, more properly, a solitary wave (sometimes the term soliton is reserved for pulses in exactly integrable nonlinear-wave models). Thus, optical solitons are localized electromagnetic waves that propagate stably in nonlinear media with dispersion, diffraction or both. In nonlinear optics, optical solitons are classified as temporal and spatial depending on whether the confinement of light occurs in time or space during wave propagation. Both types of optical solitons have been the focus of extensive research [6].

1.2.1 Temporal solitons

Temporal solitons represent optical pulses that maintain their shape during propagation. Their existence in optical fibres were predicted by Hasegawa and Tappert [7] and first observed experimentally by Mollenauer et al [8]. Physical properties of optical fibers have been studied since the 1960's, such as their waveguide characteristics, chromatic dispersion, loss, and nonlinearity [9]. When an optical pulse is transmitted in a waveguide, various Fourier components of the pulse will experience different indices of refraction and hence different Fourier components travel with different velocities called group velocity. Chromatic dispersion of optical fibers brings about the dependence of group velocity on frequency. In the presence of dispersion, different spectral components of an optical pulse propagates at different group velocities, which leads to pulse broadening. This phenomenon is referred to as group-velocity dispersion (GVD). Chromatic dispersion has two contributions: material dispersion and waveguide dispersion. Material dispersion originates from a physical property of the silica, i.e. a retarded response of bound electrons in silica for light wave electric field, which gives frequency dependence of the refractive index. Waveguide dispersion arises from the geometry of the guided structure such as a core radius which gives frequency dependence of propagation constant of a mode. Fiber loss, another limiting factor of transmission distance, originates from material absorption in the far-infrared region and Rayleigh scattering arising from random density fluctuation that takes place during fiber fabrication. Pure silica absorbs light at the wavelength $\sim 2\mu m$ while Rayleigh scattering loss is dominant at short wavelengths.

Hence the fiber exhibits a minimum loss at the wavelength in the vicinity of $1.55\mu m$. Nonlinear effects in optical fibers have two major contributions: nonlinear refractive index arising from nonlinear electric polarization of bound electrons in the silica, and stimulated inelastic scattering arising from the excitation of vibrational mode of the silica. Since silica fibers do not exhibit second order nonlinear effects due to the molecular inversion symmetry, third order nonlinear electric polarization (Kerr nonlinearity) contributes to the lowest order nonlinear effect in fibers. Since the Kerr effect induces intensity dependence of the refractive index of silica, it brings about intensity dependent phase shift. The nonlinear phase shift introduced by the optical field itself is called self phase modulation, whereas the cross phase modulation is a phase shift which is induced by the other light co-propagating with different frequencies or polarization components. These nonlinear phase shifts result in spectral broadening during propagation.

The theoretical prediction of optical solitons by Hasegawa and Tappert attracted profound interest as an ultimate solution to overcome the nonlinear effects. Hasegawa and Tappert derived the nonlinear Schrodinger equation for the propagation of the slowly varying envelope of an optical pulse in a fiber with dispersion and nonlinearity and demonstrated the soliton solutions to be effective in high-speed signal transmission in optical fibers. Optical soliton preserves its shape during propagation, where the nonlinear chirp arising from Kerr effect counteracts with the dispersion-induced chirp. Solitons thus do not suffer from either the pulse broadening in the presence of GVD or the spectral broadening due to the self phase modulation. This indicates their potential application to high-speed optical transmission. The important thing is that, theoretically speaking, the integrability of the nonlinear Schrodinger equation guarantees the stability of soliton propagation and, at the same time, indicates that any pulse of arbitrary shape launched with a proper amount of power evolves itself to a soliton (or solitons) during propagation whenever the nonlinearity is maintained large enough by loss compensated by amplifiers.

1.2.2 Pulse propagation in Optical fibres

The study of most nonlinear effects in optical fibers involves the use of short pulses with widths ranging from $\sim 10ns$ to $10fs$. When such optical pulses propagate inside a fiber, both dispersive and nonlinear effects influence their shape and spectrum. The main equation governing the evolution of optical fields in nonlinear medium is known as nonlinear Schrodinger equation.

In this section, the derivation of NLS equation for a pulse propagating in a nonlinear optic fibre is outlined. The Maxwell equations can be used to obtain the following wave(for the electric field) equation associated with an optical wave propagating in such a medium,

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2\mathbf{P}}{\partial t^2}, \tag{1.1}$$

where c is the speed of light in vacuum and ε_0 is the vacuum permittivity. The induced polarization $\mathbf{P}$ consists of two parts:

$$\mathbf{P}(\mathbf{r},t) = \mathbf{P}_L(\mathbf{r},t) + \mathbf{P}_{NL}(\mathbf{r},t) \tag{1.2}$$

where the linear part $\mathbf{P}_L$ and $\mathbf{P}_{NL}$ are related to the electric field by the general relations

$$\mathbf{P}_L(\mathbf{r},t) = \varepsilon_0 \int_{-\infty}^{\infty} \chi^{(1)}(t-t').\mathbf{E}(\mathbf{r},t')dt' \tag{1.3}$$

$$\mathbf{P}_{NL}(\mathbf{r},t) = \varepsilon_0 \int\int\int_{-\infty}^{\infty} \chi^{(3)}(t-t_1, t-t_2, t-t_3) \\ x\mathbf{E}(\mathbf{r},t_1)\mathbf{E}(\mathbf{r},t_2)\mathbf{E}(\mathbf{r},t_3)dt_1dt_2dt_3 \tag{1.4}$$

where χ^1 and χ^3 are the first and third order susceptibility tensors. These relations are valid in the electric dipole approximation under assumption that the medium response is local. The pulse envelope is time dependent and can be written as

$$\mathbf{E}(\mathbf{r},t) = A(Z,t)F(X,Y)\exp(i\beta_0 Z). \tag{1.5}$$

where F(X,Y) is the transverse field distribution associated with the fundamental mode of a single-mode fiber. The time dependence of A(Z,t) implies that all spectral components of the pulse may not propagate at the same speed inside an optical fiber because of the chromatic dispersion. This effect is included by modifying the refractive index as

$$\tilde{n} = n(\omega) + n_2|E|^2. \tag{1.6}$$

The frequency dependence of $n(\omega)$ plays an important role in the formation of temporal solitons. It leads to broadening of optical pulses in the absence of the nonlinear effects. To obtain an equation satisfied by the pulse amplitude A(Z,t), it is useful to work in the Fourier domain for including the effects of chromatic dispersion and to treat the nonlinear term as a small perturbation. The Fourier transform of $\tilde{A}(Z,\omega)$ is found to satisfy

$$\frac{\partial \tilde{A}}{\partial z} = i[\beta(\omega) + \Delta\beta - \beta_0]\tilde{A}. \tag{1.7}$$

where $\beta(\omega) = k_0 n(\omega)$ and $\Delta\beta$ is the nonlinear part defined as

$$\Delta\beta = k_0 n_2 |A|^2 \frac{\int\int_{-\infty}^{\infty} |F(X,Y)|^4 dxdy}{\int\int_{-\infty}^{\infty} |F(X,Y)|^2 dxdy}. \tag{1.8}$$

Eq.(1.7) implies that each spectral component within the pulse envelope acquires a phase shift, whose magnitude is both frequency and intensity dependent as it propagates down the fiber. Taking the inverse transform of Eq.(1.7)and obtain the propagation equation for $A(Z,t)$. Expanding β in a Taylor series around the carrier frequency ω_0 as

$$\beta(\omega) = \beta_0 + (\omega - \omega_0)\beta_1 + \frac{1}{2}(\omega - \omega_0)^2\beta_2 + ..., \tag{1.9}$$

where $\beta_m = (\frac{d^m\beta}{d\omega^m})_{\omega=\omega_0}$ Substituting Eq.(1.9) in Eq.(1.7) and taking the inverse transform, we obtain the resulting equation for A(Z,t) as

$$\frac{\partial A}{\partial Z} + \beta_1 \frac{\partial A}{\partial t} + i\beta_2 \frac{\partial^2 A}{\partial t^2} = i\gamma |A|^2 A. \tag{1.10}$$

where $\omega - \omega_0$ is replaced by the differential operator $i(\partial/\partial t)$ during the Fourier transform operation. The parameters β_1 and β_2 include the effects of dispersion to first and second orders respectively. Physically, $\beta_1 = 1/v_g$, where v_g is the group velocity associated withe pulse and β_2 takes into account the dispersion of group velocity. For this reason, β_2 is called the group velocity dispersion parameter. Eq.(1.10) can be reduced to the (1+1)dimensional NLS equation by making the following transformation variables,

$$\tau = (t - \beta_1 Z)/T_0, z = Z/L_D, u = \sqrt{|\gamma| L_D} A, \qquad (1.11)$$

where T_0 is a temporal scaling parameter and $L_D = T_0^2/|\beta_2|$ is the dispersion length. In terms of these new variables, Eq.(1.10) takes the form

$$i\frac{\partial u}{\partial z} - \frac{s}{2}\frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0, \qquad (1.12)$$

where $s = sign(\beta_2) = \pm(1)$ stands for the sign of GVD parameter. The GVD parameter β_2 can be positive or negative depending on the wavelength. The nonlinear term is positive for silica fibers but may become negative for waveguide made of semiconductor materials. Because of the two different sign of GVD parameter, optical fibers for which $n_2 > 0$, can support two different types of solitons. Eq.(1.12) has solutions in the form of dark temporal solitons in the case of normal GVD ($s = +1$) and bright temporal solitons in the case of anomalous GVD ($s = -1$). In the context of optic fibers, temporal solitons have potential applications in the field of optic fiber communications. Solitons are mainly used to increase the bit-rate or transmission capacity of the fiber by reducing the losses and dispersion effects.

Fibres with varying dispersion parameter

Soliton based communications also face some technical difficulties unique to solitons. One of the most serious issues is the phenomenon referred to as the Gordon-Haus effect [10]. Gordon-Haus effect is a timing jitter which originates from random fluctuation of the carrier frequency of solitons caused by the nonlinear interaction with amplified spontaneous emission noise. Gordon-Haus effect crucially limits the available capacity and transmission distance in soliton based systems. Nonlinear interaction between two neighboring solitons in a single channel degrades the performance [11, 12], since two solitons, when they are in-phase, attract with each other during propagation, resulting in their complete overlap at some distance. Though a number of attempts have been made to overcome these limitations, a significant breakthrough has finally come about in soliton transmissions: the discovery of dispersion managed (DM) solitons [13].

Dispersion managed solitons

In soliton systems, GVD of the fiber is assumed to remain constant along the fiber link. It turns out that soliton systems benefit considerably if the GVD parameter varies along the link length. DM soliton is a new stable nonlinear pulse which propagates in a dispersion managed fiber which implies that the local dispersion coefficient periodically alternates between positive and negative values. The first commercial fibre optic telecommunications link using the solitons was actually made use of DM solitons [14].

Dispersion decreasing fibers

The optical fibers in which the magnitude of GVD parameter decreases along the direction of propagation of optical pulses are referred to as dispersion decreasing fibers(DDFs). It can be made by tapering the core diameter of the fiber at the perform-drawing stage. As the waveguide contribution to GVD parameter depends on the core size, its value decreases along the fiber length. Such dispersion decreasing fibers were used for pulse compression in several experiments [15].

1.2.3 Spatial solitons

The study of self- guided optical beams that propagate in slab wave guides or bulk nonlinear media without supporting waveguide structures have shown great interest in recent years. Optical beams have an innate tendency to spread (diffract) as they propagate in a homogeneous unbounded medium. However this diffraction can be compensated by using refraction, if the material refractive index is increased in the transverse region occupied by the beam. Such a structure becomes an optical wave guide and confines light to the high-index region by providing a balance between diffraction and refraction. In essence, an optical beam can create its own waveguide and be trapped by this self induced waveguide. The input beam diffracts at low power but forms a spatial soliton when the intensity is large enough to create a self induced waveguide by changing the refractive index. This change is largest at the beam center and gradually reduces to zero near the beam edges, resulting in a graded index wave guide. The spatial soliton can be thought of as the fundamental mode of this waveguide. i.e., Spatial solitons originate from the dynamic balancing of two opposing tendencies, namely, the tendency for the beam to expand due to diffraction, and the tendency for the beam to contract due to self-focusing.

Formation of spatial solitons can be understood through a lens analogy. Diffraction creates a curved wavefront similar to that produced by a concave lens and spreads the beam to a wider region. The index gradient created by self- focusing effect, in contrast, acts like a convex lens that tries to focus the beam towards the beam center. i.e., a Kerr medium acts as a convex lens. The beam can become self trapped and propagate without any change in its shape if the two lensing effects cancel each other [16]. Figure 1 shows the basic concept schematically.

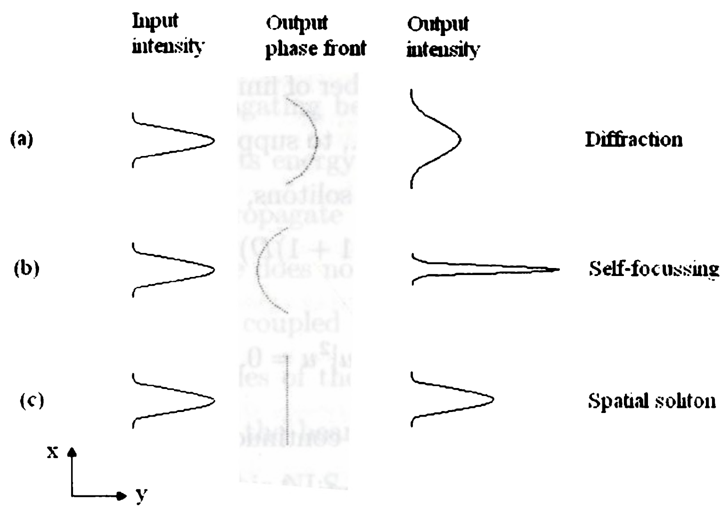

Figure 1.1: Schematic illustration of lens analogy for spatial solitons

The dimensionality of the NLS equation can change, depending on the nature of the nonlinear medium. For example, when a nonlinear medium is in the form of a planar waveguide, the optical field is confined in one of the transverse directions, say the vertical axis, by the waveguide itself. In one transverse dimension, the diffraction formally resembles the anomalous dispersion in the temporal domain therefore the intensity-dependent lens can exactly compensate diffraction, and the resulting beam may propagate without spreading or self-compression. Such a beam has the form of a stripe in a planar waveguide (the waveguide confines the beam in the other transverse direction), as was first observed by Aitchison et al (1990) [17]. Depending on its size, a planar wave guide supports a number of finite modes. It is often designed to have single moded. i.e., to support the fundamental mode alone. In analogy with fiber solitons, the normalized NLS equation for $(1+1)$dimensional $((1+1)D)$ spatial solitons can be written as

$$iu_z + \frac{1}{2}u_{xx} + |u|^2u = 0. \tag{1.13}$$

From a mathematical point of view, continuous wave beam propagation in planar waveguide is identical to the phenomenon of pulse propagation in fibres [18, 19]. i.e., the existence of stable soliton solutions which may also be regarded as (1+1)D objects, where, the first 1 refers to the transverse spatial coordinate x in this case is not surprising. This equation is exactly integrable by means of IST [20]. Generally speaking, integrability means that any localized input beam will be decomposed into stable solitary waves (or solitons) and radiation, and also that interaction of solitons is elastic. From the physical point of view, the integrable NLS equation describes (1+1)D beams in a Kerr nonlinear medium in the frame work of the so called paraxial approximation. The above equation has simple shape-preserving solution

$$u(z,x) = a sech(ax) \exp(ia^2 z/2), \tag{1.14}$$

which represents the fundamental mode of the optical waveguide induced by the propagating beam. If the input beam has the correct shape, all of its energy will be contained in this mode, and the beam will propagate without change in its shape. If the input beam shape does not exactly match the 'sech' shape some energy will be coupled into higher order bound modes or into radiation modes of the nonlinear wave guide. Though mathematically both the beam and pulse propagation can be described by same cubic NLS equation, there exists crucial difference between these two physical phenomena. In the case of pulse propagation in fibers the absolute value of GVD is small enough to be compensated by a weak nonlinearity produced by Kerr effect in optical fibers. Therefore nonlinear effects in fibres are always weak, and pulse propagation in fibres is well modeled by cubic NLS equation. In application to spatial optical solitons, the cubic NLS is not an adequate model. The optical field is confined in two transverse directions in the case of the propagation of continuous wave beams in bulk nonlinear media. In this case, much higher input powers are required to compensate for spreading due to diffraction, which is not a small effect. i.e., refractive index experiences large deviations from Kerr dependence. The number of transverse dimension D in nonlinear optical problem plays an important role in determining the stability or instability of the solutions. However, (2 + 1)D spatial solitons (self-formed cylindrical beams) in media with the Kerr nonlinearity are unstable, unlike their (1 + 1)D counterparts, because two-dimensional fluctuations may destroy the balance between the nonlinearity and diffraction in that case. In particular, an increase of the intensity leads to self-focusing of the cylindrical beam, which further increases the intensity and the corresponding intensity-dependent correction to the index of refraction, which leads to still stronger focusing and increase of the intensity, and so on. This self-accelerating process of the nonlinear self-focusing is referred to as collapse of the beam [21]. One way to avoid such a behavior and stabilize the (2 + 1)D spatial soliton is to have saturation of the nonlinearity. Accounting for these effects, lead to nonintegrable models of generalized nonlinearities, not possessing the properties of integrability and not allowing elastic soliton collisions. The generalized NLS equation takes the form,

$$iu_z + \frac{1}{2}(u_{xx} + u_{yy}) + F(|u|^2)u = 0. \qquad (1.15)$$

where the functional form of $F(|u|^2)$ is related to nonlinear refractive index. Though this equation is not integrable by the IST, it can have solitary wave solutions.

Models of non-Kerr nonlinearities

Different models have been used for the functional form of nonlinear refractive index. they can be divided into three classes, referred to as competing, saturable, and transitive nonlinearities.

Competing nonlinearities

The nonlinear refractive index of certain materials begin to deviate from the $n_2 I$ dependence for large intensities. Such deviations are observed experimentally for nonlinear materials, such as semiconductor waveguides, (e.g., AlGaAs, CdS etc.), semiconductor doped glasses, and organic polymers. The measurements for a poly diacetylene p-toluene sulphonate (PTS) crystal in wavelength region near $1600nm$ reveal that variations of the nonlinear refractive index with input intensity can be modeled by a cubic-quintic form of nonlinearity.

$$n_{nl}(I) = n_2 I + n_3 I^2, \tag{1.16}$$

where n_2 and n_3 have opposite signs. For $n_2 > 0$ but $n_3 < 0$ this form describes a competition between self-focusing occurring at low intensities and self-defocusing taking over at high intensities. The opposite occurs for $n_2 < 0$ and $n_3 > 0$. The above form can be further generalized as

$$n_{nl} = n_p I^p + n_{2p} I^{2p}, \tag{1.17}$$

where p is a positive constant and the coefficients n_p and n_{2p} have opposite signs so that $n_p n_{2p} < 0$. This model reduces to cubic-quintic nonlinearity for $p = 1$.

Saturable nonlinearities

Studies of spatial solitons have made rapid progress since the mid-1990s, when new soliton-supporting nonlinear optical interactions became available to experiments. Segev et al., had predicted that the photorefractive effect in electro-optic materials could be exploited to create a saturable nonlinear index of refraction that would support solitons [22, 23].

The nonlinear refractive index n_{nl} describing the saturable nonlinearity has the form

$$n_{nl}(I) = n_{\infty}(1 - \frac{1}{1 + I/I_{sat}^{p}}), \tag{1.18}$$

where I_{sat} the saturation intensity, n_{∞}, the maximum change in refractive index and p is a constant. This form reduces to Kerr nonlinearity for $I << I_s$ with $n_2 = n_{\infty}p/I_{sat}$. This model describes the nonlinearity of a two-level atomic system for $p = 1$.

Photorefractive solitons were observed experimentally soon afterwards and since then a variety of such solitons, of both 1D and 2D types, have been discovered and explored [24, 25]. This includes landmark advances, such as self-trapping of incoherent light and the recent generation of ordinary and vortex solitons in optically induced photonic lattices, as well as robust necklace shaped soliton clusters [26, 27, 28].

Transitive nonlinearities

Bistable solitons require a special form of intensity dependent refractive index exhibiting transition from one functional form to another as intensity increases [29]. A simple model describing transitive nonlinearity has the form

$$\begin{aligned} n_{nl}(I) &= n_{21}I, \quad I < I_{cr} \\ n_{nl}(I) &= n_{22}I, \quad I > I_{cr}. \end{aligned} \tag{1.19}$$

Bistable soliton possess attractive properties that may be useful for their application in all-optical logic and switching devices. Nonlinear optical materials with such a form of intensity dependence are not yet known.

1.2.4 Spatiotemporal dynamics

The spatial and temporal solitons are the special case of a more larger class of nonlinear phenomena in which spatial and temporal effects are coupled and occur simultaneously. When a pulsed optical beam propagates through a bulk nonlinear medium, it is affected by diffraction and dispersion simultaneously but at the same time two effects become coupled through mediums nonlinearity. Such a space time coupling leads to plethora of novel nonlinear effects, including the possibility of spatiotemporal collapse or pulse splitting and the formation of light bullets. The general form of (3+1)D NLS equation is given by

$$iu_z + \frac{1}{2}(u_{xx} + u_{yy}) - \frac{s}{2}(u_{\tau\tau} + F(|u|^2)u = 0. \tag{1.20}$$

Soliton -like solutions of this equation are called spatiotemporal solitons, since their optical field is confined in both space and time. One might naively expect that nonlinear responses suitable for the formation of a stable low dimensional soliton would also be adequate for the formation of solitons in all dimensions. That such is not the case, and, actually, that dimensionality is a central issue in the formation of solitons first of all, because of the problem of stability against the collapse in the multidimensional case. Therefore, progress in the area requires specific conceptual, analytical, numerical, and experimental advances. In summary, the quest for spatiotemporal solitons, or light bullets, faces two main challenges: first, physically relevant models of nonlinear optical systems, based on evolution equations that allow stable three-dimensional propagation, ought to be identified; second, suitable materials should be found where such models can be implemented. In this thesis we have explored the existence of stable higher dimensional spatial and spatiotemporal solitons in cubic and cubic-quintic media.

1.3 Overview

This thesis provides a theoretical study of soliton propagation in inhomogeneous single mode and coupled optic fibers and the stabilization of multidimensional spatial solitons. The content of the thesis is organized as follows. Chapter 1 is an introduction to temporal and spatial solitons. An inhomogeneous optic fibre medium with gain(loss) and frequency chirping has been analyzed within the integrable limit in chapter 2. The two soliton solution for the nonlinear Schrodinger equation has been constructed using Backlund transformation technique and a recursive method. The two soliton propagation has been studied and it finds application in soliton effect pulse compression technique. In practical applications, the integrable system analyzed in chapter2 is not an adequate model. Chapter 3 deals with the study of the nonintegrable inhomogeneous optic fibre system with varying dispersion. The NLS equation representing the dynamics of the inhomogeneous optic fibre system is not integrable and cannot be solved analytically in general. The variational analysis, a semi analytical approach, based on Lagrangian formalism of classical mechanics has been used to study the system. The inhomogeneous optic fibre system has been studied numerically using split-step Fourier method. The soliton propagation in inhomogeneous single mode optic fibers have potential application in pulse compression mechanisms in which the objective is to produce a train of ultrashort pulses.

The study has been extended to coupled systems in chapter 4. The coupled nonlinear Schrodinger equations has been studied variationally and numerically to analyze switching dynamics. We have considered a fibre coupler with cores using nonlinear optical fibres with varying dispersion and nonlinearity to analyze soliton switching. The study of the propagation of optical solitons in two mode nonlinear directional coupler with inhomogeneous dispersion and nonlinearity have great interest due to their application for all optical switching.

Spatial and spatiotemporal solitons have been suggested to be ideal candidates for a number of potential practical applications including all-optical soliton steering and switching. A challenging issue is the search for physically relevant models in which stable higher dimensional spatial solitons exist. In forth coming chapters, we have formulated and studied certain models in which stable higher dimensional spatial solitons exist. Chapter 5 deals with the study of of stabilization of two- dimensional spatial solitons in Kerr media with diffraction management and nonlinearity management. The nonlinear Schrodinger equation which governs the dynamics of two dimensional spatial solitons in Kerr media with periodically varying diffraction and nonlinearity has been analyzed in this chapter using variational approach and numerical studies. Analytical expressions for soliton parameters have been derived using variational analysis. Stability of (2+1) D spatial soliton also has been studied using Kapitsa averaging method. Variational equations and partial differential equation have been simulated numerically. The study has been extended to the stabilization of three dimensional light bullets in Kerr media in chapter 6. The stabilization of spatiotemporal solitons with periodically varying dispersion, diffraction and nonlinearity has been analyzed using variational approach and Kapitsa averaging method.

Chapter 7 presents a study of the stabilization of spatial and spatiotemporal solitons in cubic-quintic media. A diffraction managed model in cubic-quintic media has been formulated and studied for the stabilization of two dimensional spatial solitons. A dispersion managed model with diffraction management has been proposed for the stabilization of three dimensional spatiotemporal soilton in bulk cubic-quintic media. The results and conclusion of this thesis has been discussed in chapter 8.

Chapter 2

Multi Soliton propagation in an inhomogeneous optic fibre medium

2.1 Introduction

The concept of optical communication has grown and attained a status such that all future communication can make extensive use of solitons. There are many factors which affect the dynamics of optical solitons and condition for generation of optical solitons in nonuniform fibers and hence the study of pulse propagation through inhomogeneous media demands special attention as it characterizes real physical systems [30]. In a real fibre, there are many factors which affect the dynamics of solitons such as losses in the medium, inhomogeneities of the fibre etc. The inhomogeneities in the fibre are due to variation in the material parameters of the fibre medium, so that the distance between neighboring atoms is not constant throughout the fibre and or variation of the fibre geometry. Fibre losses are detrimental because they reduce peak power of solitons along the fibre length. This chapter is devoted to study of an inhomogeneous optic fibre medium with frequency chirping, which finds application in nonlinear compression of soliton pulses.

An important application of nonlinear effects in optical fibres occurs in the field of optical pulse compression. The nonlinear and dispersive effects occurring simultaneously inside silica fibres can be used to produce ultra short pulses. The basic idea behind optical pulse compression can be understood from the propagation of chirped optical pulses in a linear dispersive medium. A linear dispersive medium imposes a dispersion-induced chirp on the pulse during its propagation. If the initial chirp is in the opposite direction of that imposed by group velocity dispersion (GVD), the two tend to cancel each other, resulting in an output pulse that is narrower than the input pulse [31]. The pulse compressors based on the nonlinear effects in optical fibres can be classified into two broad categories, referred to as fibre-grating or prism compressors and soliton effect compressors [32, 33]. In a fibre grating compressor, the pulse is propagated in the normal dispersion regime of the fibre and then compressed externally using a grating pair. The role of fibre is to impose a linear, positive chirp on the pulse through a combined effect of self phase modulation (SPM) and GVD. The grating pair provides anomalous GVD required for compression of positively chirped pulses. The soliton-effect compressors make use of higher order solitons forming when SPM and anomalous GVD occur simultaneously. Optical pulses at wavelengths exceeding $1.3\mu m$ generally experience both SPM and GVD during their propagation in silica fibres. Such a fibre can act as a compressor without the need of an external grating pair and has been used for this purpose [34]. The compression mechanism in soliton-effect compressors is related to a fundamental property of higher order solitons. By an appropriate choice of the fibre length, input pulse can be compressed by a factor that depends on the soliton order N. The pulse quality using this technique is poor since the compressed pulse carries only a fraction of the input energy, while the remaining energy appears in the form of a broad pedestal. This pedestal not only leads to a deterioration in the quality of the pulse and the energy characteristics of the compression, but also makes the compressed pulse in the fibre unstable due to the nonlinear interaction of the pedestal with the compressed pulse. For this reason the compression of fundamental solitons with no pedestal component is of great interest. Another pulse compression technique makes use of optical fibres in which the magnitude of GVD parameter dispersion profile adiabatic pulse compression [35, 36].

In this chapter, an inhomogeneous optic fibre medium with varying dispersion, gain(loss) and frequency chirping has been analyzed within the integrable limit. The two soliton solution for the nonlinear Schrodinger equation has been constructed using Backlund transformation technique and a recursive method. As the two-soliton propagates through the inhomogeneous fibre, the traveling pulse gets compressed and amplified which supports multi soliton pulse compression technique.

2.2 Soliton solution

The propagation of pulse in an inhomogeneous optical fibre with varying dispersion is governed by the NLS equation

$$iq_z + p(z)q_{tt} + 2|q|^2q = F(z,t)q, \tag{2.1}$$

where $q(z,t)$ represents the complex envelope amplitude, subscripts t and z respectively denote the partial derivative with respect to normalized time and distance and $p(z)$ is the GVD parameter which varies along the fibre length. $F(z,t)$ is the inhomogeneity function related to gain for $\alpha_2(z) < 0$, loss for $\alpha_2(z) > 0$ and phase modulation which in general is a function of z and t [37] and is given by

$$F(z,t) = \alpha_1(z)t^2 - i\alpha_2(z). \tag{2.2}$$

Eq.(2.1) now becomes

$$iq_z + p(z)q_{tt} + 2|q|^2q - \alpha_1(z)t^2q + i\alpha_2(z)q = 0. \tag{2.3}$$

This equation representing an inhomogeneous optic fibre system with varying dispersion $p(z)$ and phase modulation term is not in general integrable. The integrability conditions for Eq.(2.3) can be identified through the linear eigen value problem for constant GVD parameter. When $p(z)$=1, $\alpha_1 = -\alpha_2^2$, and putting $\alpha_2 = \beta$, Eq.(2.3) becomes

$$iq_z + q_{tt} + 2|q|^2q + \beta^2t^2q + i\beta q = 0. \tag{2.4}$$

The term β^2t^2q represents frequency chirping. The complete integrability of Eq.(2.4) is confirmed by the existence of Lax pair for arbitrary values of β [38, 39]. Using a recursive method.

and Backlund transformation technique, we have constructed the two soliton solution of Eq.(2.4). A variable transformation given by

$$q(z,t) = Q(z,t)\exp\left(\frac{i\beta t^2}{2}\right), \tag{2.5}$$

is introduced to construct Lax pairs, then Eq.(2.4) becomes

$$iQ_z + Q_{tt} + 2|Q|^2Q + 2i\beta Q + 2i\beta tQ_t = 0 \tag{2.6}$$

The ZS/AKNS inverse scattering problem is defined by [40, 41, 42]

$$\psi_t = U\psi,\ \psi_z = V\psi, \tag{2.7}$$

where ψ is a two component wavefunction and U and V are Lax pairs given by

$$U = \begin{bmatrix} -i\lambda & Q \\ -Q^* & i\lambda \end{bmatrix}, \tag{2.8}$$

$$V = \begin{bmatrix} -2i\lambda^2 + 2\lambda i\beta t + i|Q|^2 & 2\lambda Q + iQ_t - 2\beta tQ \\ -2\lambda Q^* + iQ_t + 2\beta tQ^* & 2i\lambda^2 - 2\lambda i\beta t - i|Q|^2 \end{bmatrix} \tag{2.9}$$

and λ is the non isospectral parameter defined by

$$\lambda = \lambda_0 \exp(-2\beta z), \tag{2.10}$$

A one soliton solution is generated using Backlund transformation[43, 44]. Following [43], we define a pseudo potential,

$$\Gamma(n-1) = \frac{\psi_1(n-1)}{\psi_2(n-1)}. \qquad (2.11)$$

and differentiating Eq.(2.11) with respect to t, we find

$$\Gamma_t(n-1) = \frac{\psi_{1t}(n-1)\psi_2(n) - \psi_1\psi_{2t}(n-1)}{\psi_1(n-1)^2}. \qquad (2.12)$$

Substituting for ψ_{1t} and ψ_{2t} from Eq.(2.7), we get

$$\Gamma_t(n-1) = -2i\lambda\Gamma(n-1) + Q(n-1) + Q(n-1)^*\Gamma(n-1)^2. \qquad (2.13)$$

and

$$\Gamma_t^*(n-1) = 2i\lambda\Gamma^*(n-1) - Q(n)\Gamma(n-1)^{*2} - Q(n)^*. \qquad (2.14)$$

Taking complex conjugate of Eq.(2.14)

$$\Gamma_t(n-1) = -2i\lambda^*\Gamma(n-1) - Q(n)^*\Gamma(n-1)^2 - Q(n). \qquad (2.15)$$

Using Eq.(2.14) and Eq.(2.15), we obtain

$$\Gamma(n-1)^2\Gamma_t^*(n-1) - \Gamma_t(n-1) = 2i\Gamma(n-1)$$
$$(\lambda|\Gamma(n-1)|^2 + \lambda^*) + Q(n)(1 - |\Gamma(n-1)|^4) \qquad (2.16)$$

and

$$\Gamma(n-1)^2\Gamma_t^*(n-1) - \Gamma_t(n-1) = 2i\lambda\Gamma(n-1)$$
$$(\lambda^*|\Gamma(n-1)|^2 + \lambda) - Q(n-1)(1-|\Gamma(n-1)|^4) \qquad (2.17)$$

Defining $\lambda_n = \mu_n + i\nu_n$, and after straight forward manipulations, the n soliton solution $Q(n)$ can be expressed in terms of the $(n-1)$ soliton solution.

$$Q(n) = -Q(n-1) - \frac{4\Gamma(n-1)\nu_n}{1+|\Gamma(n-1)|^2}. \qquad (2.18)$$

For the trivial case $n = 1$, $Q(0) = 0$ and from Eqs. (2.7), (2.8) and (2.9), we get the eigen functions $\psi_1(0)$ and $\psi_2(0)$ as

$$\psi_1(0) = \alpha(0)\exp\left[-i(2\lambda t + \int 2\lambda^2 dz)\right], \qquad (2.19)$$

$$\psi_2(0) = \gamma(0)\exp\left[i(2\lambda t + \int 2\lambda^2 dz)\right]. \qquad (2.20)$$

Substituting Eqs.(2.19) and (2.20) in Eq.(2.11) and putting $\lambda = \mu_1 + i\nu_1$, we obtain $\Gamma(0)$ as

$$\Gamma(0) = \exp\left[-i4\mu_1 t - i4\int(\mu_1^2 - \nu_1^2)dz + 2\delta_1 + 4\nu_1 t + 8\int \mu_1\nu_1 dz + 2\Delta_1\right] \qquad (2.21)$$

where we have used $\frac{\alpha(0)}{\gamma(0)} = \exp[-2i(\delta_1 + i\Delta_1)]$. Substituting Eq.(2.21) in Eq.(2.18), we obtain one soliton solution of Eq.(2.6).

$$Q(1) = -2\nu_1 \exp[-i4\mu_1 t + 4\int(\mu_1^2 - \nu_1^2)dz + 2\delta_1]\mathrm{sech}[4\nu_1 t + 8\int \mu_1\nu_1 dz + 2\Delta_1], \tag{2.22}$$

and $q(1)$ is given as

$$q(1) = Q(1)\exp(\frac{i\beta t^2}{2}). \tag{2.23}$$

2.3 Two soliton propagation

To construct n soliton solutions, we consider a transformation ψ to ψ' and Q to Q' where unprimed quantities refer to $(n-1)$ and primed quantities denote n soliton solutions. The Bargmann result that the n soliton solution of the nonlinear Schrodinger equation can be looked upon as a potential giving n bound states [45, 46] and the relation between solitons and bound states suggests that $\psi(n)$ and $\psi(n-1)$ will differ by a linear function of λ [47]. Thus we write

$$\psi_1' = A\psi_1 + B\psi_2, \tag{2.24}$$

$$\psi_2' = C\psi_1 + D\psi_2. \tag{2.25}$$

with $A = a_1\lambda + a_0$, $B = b_1\lambda + b_0$, $C = c_1\lambda + c_0$ and $D = d_1\lambda + d_0$ where a_1, b_1, c_1 and d_1 are functions of $Q(z,t)$ and $Q^*(z,t)$ and through them functions of z and t. Differentiating Eq.(2.24) and Eq.(2.25) with respect to t and using Eq.(2.7) we obtain the following relations,

$$\begin{aligned} A_t &= Q'C + BQ^*, B_t = -2iB\lambda - AQ + Q'D, \\ C_t &= 2i\lambda C - Q'A + Q^*D, \\ D_t &= -Q^{*'}B - QC. \end{aligned} \tag{2.26}$$

Using Eqs. (2.24), (2.25) and (2.26) we obtain a set of differential equations for a, b, c and d which are easy to solve. The solutions are $a_1 = \alpha(z)$, $b_1 = 0$, $c_1 = 0$, $d_1 = \delta(z)$ with

$$\begin{aligned} b_0 &= -\frac{1}{2}i\alpha(-Q + \frac{Q'\delta}{\alpha}), \\ c_0 &= -\frac{1}{2}i\alpha[Q^* - \frac{Q^*\delta}{\alpha}], \\ d_0 &= -\frac{-\delta}{\alpha}a_0 + \gamma(z), \\ a_0 &= i\alpha[i\mu' - \frac{1}{2}(4\nu_1^2 - |Q + Q'|^2)^{\frac{1}{2}}], \end{aligned} \tag{2.27}$$

where α, γ, δ are integration constants and we have used $\frac{\delta}{\alpha} = -1$ and $\gamma = 2\alpha\mu'$. Substituting the values of A, B, C and D in terms of a_1, b_1, c_1, d_1 and a_0, b_0, c_0, d_0 from Eq.(2.27) in Eq.(2.24) and Eq.(2.25) we obtain a recurrence relation connecting n and $(n-1)$ wavefunctions.

$$\psi_1' = [-i\lambda + i\mu' - \frac{1}{2}(4\nu'^2 - |Q+Q'|^2)^{\frac{1}{2}}]\psi_1 + \frac{1}{2}(Q+Q')\psi_2, \qquad (2.28)$$

$$\psi_2' = [i\lambda - i\mu' - \frac{1}{2}(4\nu^{2'} - |Q+Q'|^2)^{\frac{1}{2}}]\psi_2 + \frac{-1}{2}(Q^* + Q^{*'})\psi_1. \qquad (2.29)$$

We know that for the trivial case, $n = 1$, $Q = 0$ and Q' is given by Eq.(2.22). Substituting these in Eq.(2.28) and Eq.(2.29) we obtain $\psi_1(1)$ and $\psi_2(1)$ which are given by

$$\psi_1(1) = [-i\lambda + i\mu_1 - \nu_1 \tanh A_1]\psi_1(0) \nu_1 \exp[-iM_1]\text{sech}A_1\psi_2(0), \qquad (2.30)$$

$$\psi_2(1) = \nu_1 \exp[iM_1]\text{sech}A_1\psi_1(0) + [i\lambda - i\mu_1 - \nu_1 \tanh A_1]\psi_2(0), \qquad (2.31)$$

where

$$M_1 = 4\mu_1 t + 4\int(\mu_1^2 - \nu_1^2)dz + 2\delta_1, \qquad (2.32)$$

$$A_1 = 4\mu_1 t + 8\int \mu_1\nu_1 dz + 2\delta_1. \qquad (2.33)$$

Now for $n = 2$, substituting Eq.(2.30) and Eq.(2.31) in Eq.(2.11) we obtain the pseudo potential $\Gamma(1)$ as

$$\Gamma(1) = \frac{P - iN}{R + iS}, \qquad (2.34)$$

where

$$P = \exp[A_2]\cos M_2(\nu_2 - \nu_1 \tanh A_1) - \Delta\mu \exp[A_2]\sin M_2 - \nu_1 \mathrm{sech} A_1 \cos M_1 \qquad (2.35)$$

$$N = \exp[A_2]\sin M_2(\nu_2 - \nu_1 \tanh A_1) + \Delta\mu \exp[A_2]\cos M_2 - \nu_1 \mathrm{sech} A_1 \sin M_1 \qquad (2.36)$$

$$R = \nu_1 \exp[A_2]\mathrm{sech} A_1 \cos(M_1 - M_2) - (\nu_2 + \nu_1 \tanh A_1), \qquad (2.37)$$

$$S = \nu_1 \exp[A_2]\mathrm{sech} A_1 \sin(M_1 - M_2) + \Delta\mu. \qquad (2.38)$$

Substituting Eq.(2.34) and Eq.(2.22) in Eq. (2.18) we obtain $Q(2)$ as

$$Q(2) = \frac{X}{Y}. \qquad (2.39)$$

Substituting Eq.(2.39) in Eq.(2.5)the two soliton solution is obtained as

$$q(2) = \frac{X \exp(\frac{i\beta t^2}{2})}{Y}, \tag{2.40}$$

where

$$X = 2\nu_1 \mathrm{sech} A_1 \exp[-iM_1][\nu_1^2 + (\Delta\mu)^2 - \nu_2^2 + 2i\Delta\mu\nu_2 \tanh A_2]$$
$$-2\nu_2 \mathrm{sech} A_2 \exp[-iM_2][-(\Delta\mu)^2 - \nu_2^2 + \nu_1^2 + 2i\Delta\mu\nu_1 \tanh A_1],$$

$$Y = \nu_1^2 + \nu_2^2 + (\Delta\mu)^2 - 2\nu_1\nu_2 \tanh A_1 \tanh A_2$$
$$-2\nu_1\nu_2 \mathrm{sech} A_1 \mathrm{sech} A_2 \cos(M_1 - M_2), \tag{2.41}$$

where $\Delta\mu = \mu_2 - \mu_1$. μ_2 and ν_2 are the parameters of the two soliton solution. By continuing this process, higher order soliton solutions can be determined. Figure (2.1) illustrates the interaction of two soliton pulses where the pulses collide at $z = 0$ and undergo compression. After collision, they traverse the length of the fibre and forced to separate further. Figures 2.2-2.4 show the propagation of pulses after interaction. After interaction, as the two soliton propagates through the fibre, its spectrum begins to broaden as a result of self phase modulation effect. Due to the joint action of self phase modulation effects and negative group velocity dispersion effects the pulse begins to be compressed forming an intense pulse even after compensating loss as shown in Figure 2.4. The additional chirping potential helps to suppress the pedestal. When two soliton pulses with a phase difference propagates, the slopes of trailing edge of first pulse and leading edge of second pulse are increased in magnitude and the frequency chirping is enhanced. Since enhanced chirping is more than sufficient to balance the broadening due to dispersion. the two pulses are effectively forced to separate further. Experiments by Tai et.al [48], Daniov et.al [49] and Suzuki et.al [50] showed that multi-soliton pulses can be compressed. Soliton-effect compression can achieve high compression rate using shorter compression lengths. Ahamed et.al [51] experimentally compressed a $3.6ps$ pulse down to $185fs$ by propagation along $30m$of the fibre.

2.4 Conclusion

An inhomogeneous optic fibre system with frequency chirping within the integrable limit has been analyzed in this chapter. The exact two soliton solution for the nonlinear Schrodinger equation has been constructed using a recursive method and Backlund transformation technique. As the pulse propagates

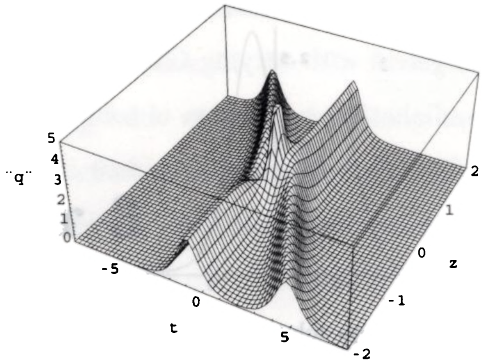

Figure 2.1: Two soliton solution of (2.40) for $\nu_1(0) = 0.9$, $\nu_2(0) = 0.9$, $\mu_1(0) = 0.5$, $\mu_2(0) = 0.08$, $\beta = .02$.

through the fibre, the pulse gets amplified and compressed which supports multi soliton pulse compression technique. The drawback of this technique is that the required input powers are high and cannot be obtained directly from semiconductor lasers and pulse quality is poor. For this reason, the compression of fundamental soliton using fibres with varying dispersion is of great interest. The inhomogeneous optic fibre system with varying dispersion is not in general integrable. The nonintegrable inhomogeneous optic fibre with varying dispersion has been studied in the next chapter.

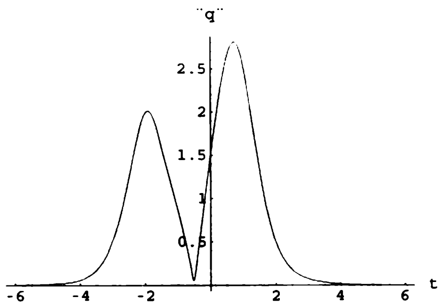

Figure 2.2: Two soliton solution at $z = 2.5$ for the same parameters as in figure(1).

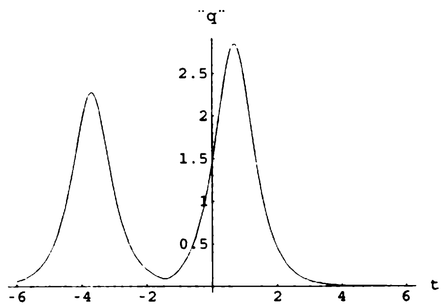

Figure 2.3: Two soliton propagation at $z = 3.5$ for the same parameters as in figure(1)

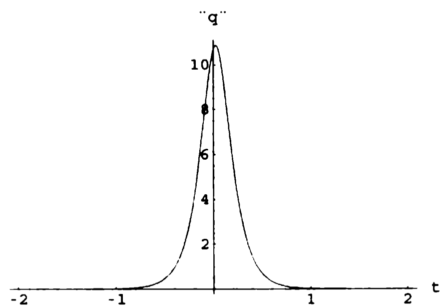

Figure 2.4: compressed soliton pulse at $z = 20$ for the same parameters as in figure(1).

Chapter 3

Soliton compression in inhomogeneous fibres with varying dispersion

3.1 Introduction

In an inhomogeneous fibre, when there is a variation in the core diameter, the group velocity dispersion (GVD) coefficient cannot be considered as a constant, but as a function of distance. For optical solitons, a small variation in dispersion has a perturbative effect similar to amplification or loss. By exploiting the property of variable GVD in good use, it will be possible to control solitons in soliton communication systems. The inhomogeneous optic fibre system with varying GVD parameter has been studied in this chapter. The study of pulse propagation in an optic fibre system with varying GVD parameter finds application in the adiabatic compression of solitons. In nonlinear fibre optics, nonlinear compression of pulses is found to have wide applications in optical telecommunication and switching purposes.

A novel technique of high quality fundamental soliton compression can be achieved using fibres with a slowly decreasing value of second order dispersion along the fibre length [52, 53, 54]. An optical fibre in which the magnitude of the GVD parameter decreases along the direction of propagation of optical pulses is referred to as a dispersion decreasing fibre (DDF) and can be made by tapering the core diameter of the fibre at the manufacturing stage. The dispersion variation in the DDF is sufficiently gradual, soliton compression is an adiabatic process such that input pulse can be ideally compressed as it propagates by retaining its soliton character. The analogy between a fibre amplifier and a DDF can be established mathematically using nonlinear Schrodinger equation (NLSE). The effect of decreasing dispersion is mathematically equivalent to adding a gain term to NLSE. The effective gain coefficient is related to rate at which GVD decreases along the fibre. This property is used to study pulse compression in dispersion decreasing fibres. The

DDF pulse-compression mechanism has been used to generate a train of ultra short pulses [55, 56, 57]. Using the fibres with a slowly decreasing value of dispersion along the fibre length enables to realize effective amplification [58, 59]. It has been found that there exists a quasi stationary pulse known as dispersion managed soliton in periodically dispersion compensated systems with features such as the reduced timing jitter caused by amplifier noise (Gordon - Haus effect) or the improved signal to noise ratio which is not present in a classical soliton system [60]. A periodically stationary pair of adjacent pulses propagating in a dispersion managed line has been recently found by the numerical averaging method and named a bisoliton [61, 62].

This chapter deals with the study of soliton propagation in an inhomogeneous optic fibre system with varying dispersion. i.e, propagation in an optic fibre with gain (loss), phase modulation and varying dispersion. The NLSE representing the dynamics of this inhomogeneous optic fibre system is not integrable and cannot be solved analytically in general. The variational analysis, a semi analytical approach, based on Lagrangian formalism of classical mechanics has been used to study the system. The inhomogeneous optic fibre system has been studied numerically using split-step Fourier method. The study has shown that as the pulse propagates through the inhomogeneous optic fibre, its width decreases and amplitude increases, which is similar to the case of adiabatic compression of solitons.

3.2 Lagrangian formulation

Propagation of optical pulses in inhomogeneous fibres with varying dispersion can be described by nonlinear Schrodinger equation of the form,

$$iq_z + p(z)q_{tt} + 2|q|^2 q - \alpha_1(z)t^2 q + i\alpha_2(z)q = 0. \tag{3.1}$$

This equation representing an inhomogeneous optic fibre system with varying dispersion $p(z)$ and phase modulation term is not integrable because of the inhomogeneity due to the varying dispersion. Thus we solve the problem approximately by means of the variational method based on the observation that the system described by Eq.(3.1) supports a well-defined solution of chirped pulse whose shape is close to Gaussian. Inorder to introduce the variational method, we start from the Lagrangian formulation of the problem [63]. Suppose we have a functional $\mathcal{L}$ called Lagrangian density which is a function of the field quantity q. and q^*. and their derivatives with respect to z and t. We adopt formulation of the infinite dimensional Euler-Lagrange problem .

For infinite dimensional problems, Lagrange's principle of least action is given by

$$\delta I = \delta \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathcal{L}(q, q^*, q_z, q_z^*, q_t, q_t^*, z, t) dtdz = 0. \tag{3.2}$$

For the variation of field q, the variation of $\mathcal{L}$ is expressed as

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial q}\delta q + \frac{\partial\mathcal{L}}{\partial q_z}\delta q_z + \frac{\partial\mathcal{L}}{\partial q_t}\delta q_t \frac{\partial\mathcal{L}}{\partial q^*}\delta q^* + \frac{\partial\mathcal{L}}{\partial q_z^*}\delta q_z^* + \frac{\partial\mathcal{L}}{\partial q_t^*}\delta q_t *. \tag{3.3}$$

Then Eq.(3.2) becomes

$$\begin{aligned}\delta I = &\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial\mathcal{L}}{\partial q}\delta q dtdz + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial\mathcal{L}}{\partial q_z}\delta q_z dtdz \\ &+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial\mathcal{L}}{\partial q_t}\delta q_t dtdz + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial\mathcal{L}}{\partial q^*}\delta q^* dtdz \\ &+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial\mathcal{L}}{\partial q_z^*}\delta q_z^* dtdz + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial\mathcal{L}}{\partial q_t^*}\delta q_t * dtdz. \end{aligned}\tag{3.4}$$

Integrating by parts,

$$\begin{aligned}\delta I = &\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial\mathcal{L}}{\partial q}\delta q dtdz - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial}{\partial z}(\frac{\partial\mathcal{L}}{\partial q_z})\delta q dtdz \\ &- \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial}{\partial t}(\frac{\partial\mathcal{L}}{\partial q_t})\delta q dtdz + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial\mathcal{L}}{\partial q^*}\delta q^* dtdz \\ &- \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial}{\partial z}(\frac{\partial\mathcal{L}}{\partial q_z^*})\delta q dtdz - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial}{\partial t}(\frac{\partial\mathcal{L}^*}{\partial q_t})\delta q^* dtdz \end{aligned}\tag{3.5}$$

i.e,

$$\delta I = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[\frac{\partial \mathcal{L}}{\partial q} - \frac{\partial}{\partial z}(\frac{\partial \mathcal{L}}{\partial q_z}) - \frac{\partial}{\partial t}(\frac{\partial \mathcal{L}}{\partial q_t})]\delta q dt dz$$
$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[\frac{\partial \mathcal{L}}{\partial q^*} - \frac{\partial}{\partial z}(\frac{\partial \mathcal{L}}{\partial q_z^*}) - \frac{\partial}{\partial t}(\frac{\partial \mathcal{L}}{\partial q_t^*})]\delta q^* dt dz = 0 \quad (3.6)$$

where the variations δq and δq^* vanish at the boundary of the integration. Since the variations δq and δq^* are taken to be arbitrary and independent in Eq.(3.6) we obtain

$$\frac{\partial \mathcal{L}}{\partial q} - \frac{\partial}{\partial z}(\frac{\partial \mathcal{L}}{\partial q_z}) - \frac{\partial}{\partial t}(\frac{\partial \mathcal{L}}{\partial q_t}) = 0, \quad (3.7)$$

and

$$\frac{\partial \mathcal{L}}{\partial q^*} - \frac{\partial}{\partial z}(\frac{\partial \mathcal{L}}{\partial q_z^*}) - \frac{\partial}{\partial t}(\frac{\partial \mathcal{L}}{\partial q_t^*}) = 0. \quad (3.8)$$

These are Euler Lagrange equations in infinite dimension. The nonlinear Schrodinger Eq.(2.3) is formulated as Euler -Lagrange equation of the form,

$$\frac{\delta \mathcal{L}}{\delta q^*} = \frac{\partial \mathcal{L}}{\partial q^*} - \frac{\partial}{\partial z}(\frac{\partial \mathcal{L}}{\partial q_z^*}) - \frac{\partial}{\partial t}(\frac{\partial \mathcal{L}}{\partial q_t^*}) = 0. \quad (3.9)$$

with the Lagrangian density generating Eq.(3.1)is

$$\mathcal{L} = \frac{i}{2}(qq_z^* - q^*q_z) + p(z)|q_t|^2 - |q|^4 - \alpha_1 t^2|q|^2 - i\alpha_2|q|^2. \quad (3.10)$$

3.3 Variational analysis

This approximate analysis originally proposed by Anderson [64] is a Ritz optimization procedure. Inorder to analyze the propagation of pulses with varying dispersion in an inhomogeneous optic fibre using variational approach, we have to assume a trial function.

The solution of Eq.(3.1) is chosen as Gaussian:

$$q(z,t) = A(z)\exp(\frac{-t^2}{2a(z)^2} + ib(z)t^2 + i\phi(z)), \tag{3.11}$$

where $A(z)$, $a(z)$, $b(z)$, $\phi(z)$ respectively describe complex amplitude, pulse width, frequency chirp and phase of the soliton. Inserting this trial function into the Lagrangian density and integrating $\mathcal{L}$ over t with this ansatz we obtain reduced Lagrangian. i.e,

$$\langle L\rangle = \int_{-\infty}^{\infty} \mathcal{L}dt, \tag{3.12}$$

and it is given by

$$\begin{aligned}\langle L\rangle = \frac{i}{2}(AA_z^* - A^*A_z)a\sqrt{\pi} + |A|^2\phi_z a\sqrt{\pi} - i\alpha_2|A|^2 a\sqrt{\pi} \\ + |A|^2 a^3(b_z - \alpha_1)\frac{1}{2}\sqrt{\pi} - |A|^4 a\frac{\sqrt{\pi}}{2} \\ +|A|^2 a^3\frac{\sqrt{\pi}}{2}p(z)(\frac{1}{a^4} + 4b^2)\end{aligned} \tag{3.13}$$

The original infinite dimensional problem Eq.(3.1) is now reduced to a Lagrangian form in finite dimension whose Lagrangian is given by Eq.(3.13) which involves only a finite set of variables.

The Euler-Lagrange equation for a finite dimensional problem can also be obtained in a way similar to the derivation for the finite dimensional case. i.e,

$$\delta I = \int_{-\infty}^{\infty} \langle L(r, r_z, z)\rangle dz = 0. \qquad (3.14)$$

where r is one of the parameters in reduced Lagrangian given by Eq.(3.13). The Euler-Lagrange equation obtained from reduced Lagrangian is given by

$$\frac{\partial\langle L\rangle}{\partial r} - \frac{\partial}{\partial z}(\frac{\partial\langle L\rangle}{\partial r_z}) = 0. \qquad (3.15)$$

Substituting r=$a(z)$, $b(z)$, $\phi(z)$ in Eq.(3.15)the following set of equations can be derived.

$$a_z = 4abp(z), \qquad (3.16)$$

$$b_z = p(z)(\frac{1}{a^4} - 4b^2) + \alpha_1 - \frac{N}{a^3\sqrt{2\pi}}, \qquad (3.17)$$

$$N = |A|^2 a\sqrt{\pi}, \qquad (3.18)$$

where N is the conserved quantity associated with the number of particles [65]. Eq.(3.16) and Eq.(3.17) are the expressions for pulse width and chirp respectively. A closed form evolution equation for pulse width is

$$a_{zz} = 4\alpha_1 a p(z) + \frac{4p(z)^2}{a^3} - \frac{Np(z)}{a^2\sqrt{2\pi}} + \frac{a_z p_z}{p(z)}. \tag{3.19}$$

That is,

$$\frac{d}{dz}[\frac{1}{p(z)}\frac{da}{dz}] = -\frac{\partial U}{\partial a}. \tag{3.20}$$

where

$$U(a,z) = \frac{2p(z)}{a^2} - 2\alpha_1 a^2 - \frac{N}{a\sqrt{2\pi}}, \tag{3.21}$$

The Hamiltonian of the system is

$$H(a, a_z, z) = \frac{a_z^2}{2p(z)} + U(a, z). \tag{3.22}$$

The soliton width evolution can be considered as the motion of a particle of mass $\frac{1}{p(z)}$ in an anharmonic potential $U(a,z)$ [66]. The ordinary differential equations given by Eq.(3.16) and Eq.(3.17) are equations of motion of the particle described by new coordinates a and b.

3.4 Numerical studies

We have solved Eq.(3.16) and Eq.(3.17) numerically to study the dynamics of soliton pulse for different dispersion decreasing GVD parameters. Figure 3.1 shows the variation of pulse width with distance for exponential profile. The phase portrait of variational equations for exponential GVD profile is as shown in Figure 3.2. When GVD profile is Gaussian, the width variation is as shown in Figure 3.3 and Figure 3.4 illustrates the phase portrait. The variation of pulse width and phase portrait using hyperbolic GVD profile is as shown in Figures 3.5-3.6. As the pulse propagates through the fibre, its width decreases and the frequency chirp increases and there is pulse compression and amplification. The pulse compression and amplification is mainly due to the interplay between the inherent gain of the dispersion decreasing profile and the effective phase modulation. It is evident from the plots that pulse compression is more in the case of Gaussian and hyperbolic profile than exponential profile. Dispersion decreasing fibres were used for pulse compression in several experiments to generate a train of ultra short pulses at high repetition rates [67, 68, 69]. Such sources of ultra short optical pulses are useful for fibre-optic communication systems.

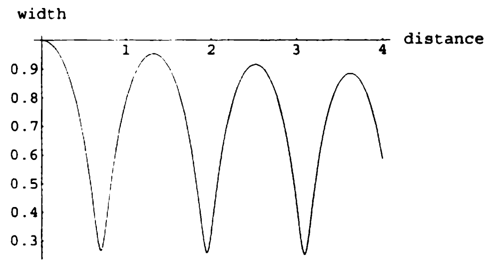

Figure 3.1: Variation of width of the pulse with propagation distance for exponential GVD profile. $\alpha_1 = 1$, $a(0) = 1$, $b(0) = 0$.

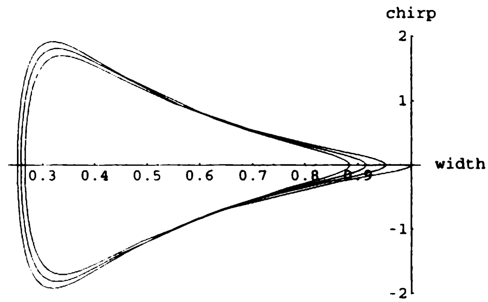

Figure 3.2: Phase portrait of variational equations for exponential GVD profile.

3.5 Split-step Fourier method

The nonlinear Schrodinger equation is a nonlinear partial differential equation which does not lend itself to analytic solutions and hence numerical approach is used to verify the results of variational analysis. The numerical methods generally used for this purpose can be classified into two broad categories known as the finite -difference methods and the pseudo spectral meth-

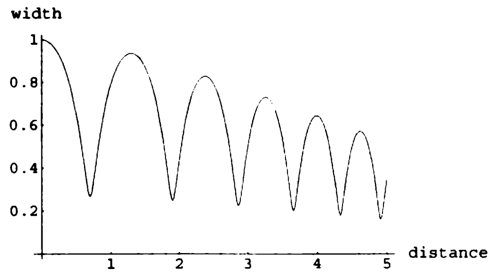

Figure 3.3: Variation of width of the pulse with propagation distance for Gaussian GVD profile. $\alpha_1 = 1$, $a(0) = 1$, $b(0) = 0$.

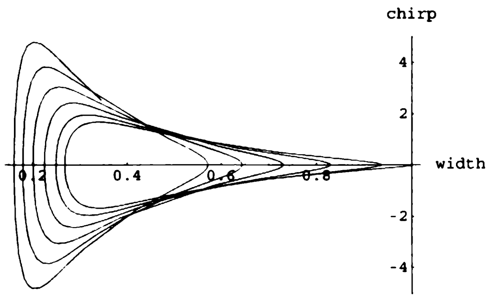

Figure 3.4: Phase portrait of variational equations for Gaussian GVD profile.

ods [70, 71, 72]. In general pseudo spectral methods are faster than the finite difference method. The one method that has been used extensively to solve the pulse-propagation problem in nonlinear dispersive media is the split-step Fourier method.

We have solved Eq.(3.1) numerically for different GVD profiles using split-step Fourier method (SSFM) [73]. The nonlinear

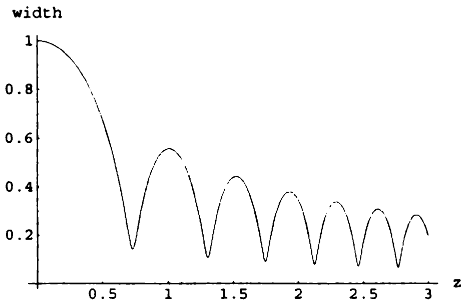

Figure 3.5: Variation of width of the pulse with propagation distance for hyperbolic GVD profile $\alpha_1 = 1$, $a(0) = 1$, $b(0) = 0$.

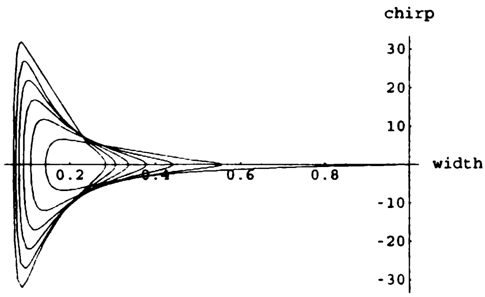

Figure 3.6: Phase portrait of variational equations for hyperbolic GVD profile.

equation can be represented in the form

$$q_z = N(q), \tag{3.23}$$

where

$$N(q) = A(q) + B(q). \tag{3.24}$$

The solution of Eq.(3.23) can be written as

$$q(z) = \exp(Nz)q_0. \tag{3.25}$$

i.e,

$$q(z) = \exp[(A + B)z]q_0. \tag{3.26}$$

The differential operator that accounts for dispersion term is now represented by the operator A and B is a nonlinear operator that governs the effect of fibre nonlinearities on pulse propagation. In general dispersion and nonlinearity act together along the length of the fibre. The split-step Fourier method obtains an approximate solution by assuming that in propagating the optical field over a small distance δ, the dispersive and nonlinear effects act independently. i.e, propagation from z to $z + \delta$ is carried out in two steps. In the first step, the nonlinearity acts alone, and $A = 0$. In the second step, dispersion acts alone and $B = 0$. Hence we can write solution in the form

$$q(z + \delta, t) = \exp(\delta A)\exp(\delta B)q(z, t), \tag{3.27}$$

The exponential operator $\exp(\delta A)$ can be evaluated in Fourier domain using

$$\exp(\delta A)q_1(z,t) = F_t^{-1}\exp[\delta A(i\omega)]F_t q_1(z,t), \qquad (3.28)$$

where F_t denotes the Fourier transform operation, $A(i\omega)$ is obtained by replacing the differential operator $\partial/\partial t$ by $i\omega$ and 'ω'is the frequency in the Fourier domain. The accuracy of split step Fourier method can be improved by adopting second order split-step method.

The accuracy of the the split-step Fourier method can be improved by adopting second order split-step method.

$$\exp[(A+B)\delta] = \exp(A\delta/2)\exp(B\delta/2)\exp(B\delta/2)\exp(A\delta/2). \qquad (3.29)$$

i.e.,

$$\exp[(A+B)\delta] \approx \exp(A\delta/2)\exp(B\delta)\exp(A\delta/2). \qquad (3.30)$$

where the fibre length δ has been split into two sections of length $\delta/2$ and the operators applied in a different order in each section. The parameters used are $\alpha_1 = 1$, $\alpha_2 = 0.02$. Numerical plots for the solution of partial differential Eq.(3.1) for different GVD parameters are as shown in Figures 3.7-3.9. The numerical plots also reveal that there is pulse compression and amplification when the pulse propagates through an inhomogeneous optic fibre medium with varying GVD parameter.

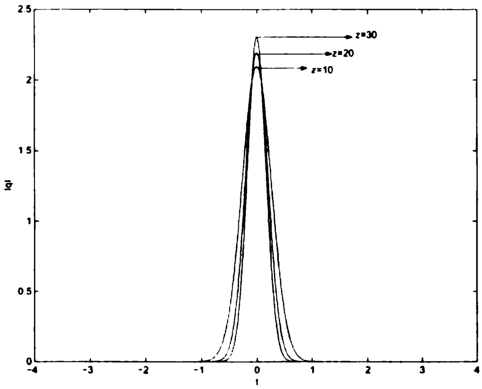

Figure 3.7: Numerical solution for partial differential equation (2.3) for Gaussian GVD profile for $\alpha_1 = 1$, $\alpha_2 = 0.02$.

3.6 Conclusion

The inhomogeneous optic fibre system has been studied for different dispersion decreasing GVD parameters using variational analysis and numerical methods. The different GVD parameters chosen are exponential, Gaussian and hyperbolic. As the pulse propagates through the fibre, the pulse gets compressed and amplified, which is similar to the adiabatic compression of solitons. Numerical and variational studies reveal that pulse compression is more in the case of Gaussian and hyperbolic GVD profiles

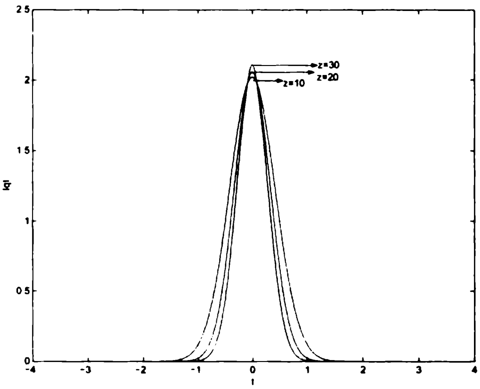

Figure 3.8: Numerical solution for partial differential equation (2.3) for exponential GVD profile for $\alpha_1 = 1$, $\alpha_2 = 0.02$.

than exponential profile. The pulse compression and amplification is mainly due to the interplay between the inherent gain of the dispersion decreasing profile and the effective phase modulation. The pulse compression for Gaussian and hyperbolic profile is very promising and can be used for the generation of ultra short pulses while DDFs with exponential dispersion profile can be used for transmission of ultrashort pulses over relatively long lengths [74]. In a constant dispersion fibre, solitons broaden as they loose energy because of weakening of nonlinear effects. The width of a fundamental soliton can be maintained inspite of fibre losses, if GVD decreases exponentially.

Soliton propagation in nonuniform single mode fibres has been studied in these chapters finds application in soliton pulse

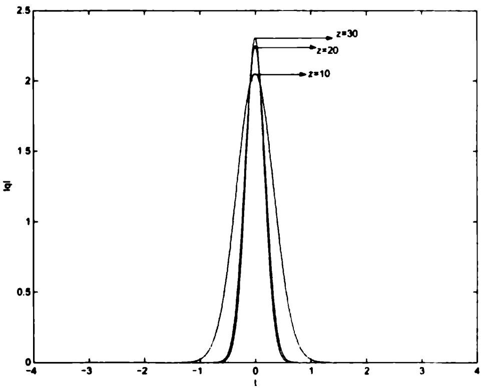

Figure 3.9: Numerical solution for partial differential equation (2.3) for hyperbolic GVD profile for $\alpha_1 = 1$, $\alpha_2 = 0.02$.

compression . The inhomogeneous optic fibre system with varying GVD parameter is nonintergrable and it finds application in the adiabatic compression of solitons while inhomogeneous optic fibre system within the integrable limit can be used in the multisoliton pulse compression technique. The study has been extended to coupled systems in the next chapter.

Chapter 4

Soliton switching in nonlinear directional coupler

4.1 Introduction

The nonlinear Schrodinger equation (NLSE) governs the dynamics of propagation of soliton pulses in a single mode fibre in anomalous dispersion regime. Though NLSE adequately describes the propagation in single-mode wave guides, routing and switching operations which involve soliton pulses require the interaction between two or more modes. When two parallel waveguides coupled through evanescent fields overlap or the coupling of two polarization modes in uniform guides are the situations of physical interest that can be described by coupled NLSE. A dual-core fibre, designed to have two cores close to each other throughout its length, can act as a fibre coupler. Their function is to split coherently an optical field, incident on one of the input ports, and direct the two parts to the output ports. Since output is directed in two different directions, such devices are also referred to as directional couplers. An important application of fibre couplers consists of using them for all optical switching. This chapter analyzes the propagation of optical solitons in two mode nonlinear directional coupler with inhomogeneous dispersion and nonlinearity.

All optical switching devices have been studied extensively since they can potentially operate at speeds much higher than those possible with electronic or optoelectronic switches. One of the most frequently studied system is a nonlinear directional coupler (NLDC), which is a special device whose transmission characteristics depend sensitively on the light intensity [75, 76]. Coupled-mode theory is used for the mathematical description of directional couplers. When the pulses are sufficiently wide, the group velocity dispersion may be neglected, and all frequency components of the pulse have a common group velocity. This is referred to as the quasi-continuous wave case. There is an obvious problem with the use of optical pulses in the quasi-continuous wave regime. Only the central intense part of an input pulse is switched since pulse wings exhibit the low power behavior. Thus a nonuniform intensity profile of optical pulses leads to distortion even when the effects of GVD are negligible and this pulse distortion is accompanied by degradation in the switching behavior [77]. Due to the remarkable shape preserving property of solitons, using solitons for switching in nonlinear directional couplers has attracted much attention and have been studied [78, 79, 80, 81, 82, 83].

Fibre couplers are called symmetric when their cores are identical in all respects. However this is rarely the case in practice. The couplers with dissimilar cores are called asymmetric couplers. The deliberate asymmetric couplers have attracted attention in recent years [84, 85]. The asymmetry includes mismatch in phase-velocity and group-velocity, differences in dispersion coefficients and effective core areas. Interesting results have been obtained for asymmetric couplers with a phase-velocity difference between the cores [86, 87]. Differences in the group velocity dispersion (GVD) parameters result from the waveguide contribution to GVD that depends on the core size. Small changes in the core shape and size can induce large changes in dispersion coefficient of the fibre operating near the zero dispersion wavelength. When the two cores in asymmetric coupler are so widely different that their dispersion coefficients have opposite signs especially the case in which the GVD is normal in the second core has been studied and [88]. Fibres with inhomogeneous dispersion have attracted attention recently due to their potential applications. They have been used for pulse compression [89], formation of soliton out of a non-soliton pulse [90], and improvement of soliton amplification in long communication lines [91].

In this chapter, we have considered a fibre coupler with cores using nonlinear optical fibres with inhomogeneous dispersion and nonlinearity to analyze soliton switching. The soliton dynamics in NLDC are governed by coupled nonlinear Schrodinger equation. The coupled differential equations for the parameters which describe switching dynamics have been derived using variational analysis. The variational equations and the partial differential equations have been solved numerically for periodically varying dispersion parameters in two cores.

4.2 Mathematical model

consider a nonlinear directional coupler made of optical fibre with inhomogeneous dispersion. The system of coupled nonlinear partial differential equations, governing the pulse dynamics is given by,

$$iu_z + \frac{d_1(z)}{2}u_{tt} + |u|^2 u + kv = 0 \quad (4.1)$$

$$iv_z + \frac{d_2(z)}{2}v_{tt} + d_n|v|^2 v + ku = 0 \quad (4.2)$$

where u and v represent the slowly varying light field envelops and the subscripts z and t correspond to partial derivatives with respect to propagation distance and retarded time respectively. The parameter $d_1(z)$ and $d_2(z)$ represent varying dispersion in the first and second core respectively and $d_n = \gamma_2/\gamma_1$ account for differences in effective core areas. The constant k stands for linear coupling.

In general the coupled nonlinear Schrodinger equations cannot be solved analytically. Particle like switching of solitons suggests the use of variational technique which offers considerable physical insight. The variational method was developed by Anderson and used for solving these coupled equations [92].

In Lagrangian formulation, Eqs.(4.1) and (4.2) are generated from the Lagrangian density,

$$L = \frac{i}{2}(uu_z^* - u^*u_z) - \frac{1}{2}|u_t|^2 + \frac{1}{2}|u|^4 \quad (4.3)$$
$$+\frac{i}{2}(vv_z^* - v^*v_z) - \frac{d(z)}{2}|v_t|^2 + \frac{d_n}{2}|v|^4 + k(u^*v + v^*u).$$

The appropriate functional form of solution is chosen as

$$u(z,t) = A_1(z) sech(t/a(z)) \exp(i\theta_1)$$
$$v(z,t) = A_2(z) sech(t/a(z)) \exp(i\theta_2). \qquad (4.4)$$

where $A_1(z)$, $A_2(z)$, $\theta_1(z)$, and $\theta_2(z)$ are amplitude and phase of soliton propagating in core1 and core2 respectively. The width of soliton's components which are linearly coupled to each other do not significantly differ and is represented as a(z) [93]. Substituting Eq.(4.4) into the expression for Lagrangian density, and integrating over t, the effective Lagrangian is obtained as

$$L_{eff} = 2a(z)(A_1(z)^2\theta_{1z} + A_2(z)^2\theta_{2z}) - \frac{1}{3a(z)}(A_1(z)^2 + d(z)A_2(z)^2) \qquad (4.5)$$
$$+\frac{2a(z)}{3}(A_1(z)^4 + d_n A_2(z)^4) + 4a(z)kA_1(z)A_2(z)\cos(\theta_1(z) - \theta_2(z)).$$

The Euler-Lagrange equation given by

$$\frac{\partial L_{eff}}{\partial p} - \frac{d}{dz}(\frac{\partial L_{eff}}{\partial p_z}) = 0, \qquad (4.6)$$

with $p = A_1(z)$, $A_2(z)$, $\theta_1(z)$, $\theta_2(z)$ yields a set of four coupled nonlinear ordinary differential equations.

$$A_{1z} = -kA_2(z)\sin(\theta_1(z) - \theta_2(z)) + \frac{A_1(z)a(z)}{2}a_z \qquad (4.7)$$

$$A_{2z} = kA_1(z)\sin(\theta_1(z) - \theta_2(z)) + \frac{A_2(z)a(z)}{2}a_z \qquad (4.8)$$

$$\theta_{1z} = \frac{1}{6a(z^2)} - \frac{2}{3}A_1(z)^2 - \frac{kA_2(z)}{A_1(z)}\cos(\theta_1(z) - \theta_2(z)) \qquad (4.9)$$

$$\theta_{2z} = \frac{d(z)}{6a(z^2)} - \frac{2d_n}{3}A_2(z)^2 - \frac{kA_1(z)}{A_2(z)}\cos(\theta_1(z) - \theta_2(z)) \qquad (4.10)$$

These four coupled equations are reduced to two coupled equations by changing variables. Introducing new variables, $\Delta(z) = A_1(z)^2 - A_2(z)^2$, $\theta(z) = \theta_1(z) - \theta_2(z)$, and using Eqs.(4.7) and (4.8) we obtain

$$\Delta_z = -2k\sqrt{(E^2 - \Delta(z)^2)}\sin(\theta(z) + a(z)\Delta(z)a_z \qquad (4.11)$$

where $E = A_1^2 + A_2^2$. Using Eqs.(4.9) and (4.10), and setting $d_n = 1$ the equation for relative phase variable is obtained as

$$\theta_z = \frac{1}{6a(z)^2}(1 - d(z)) - \frac{2\Delta(z)}{3} + \frac{k\Delta(z)cos\theta(z)}{\sqrt{(E^2 - \Delta(z)^2)}} \qquad (4.12)$$

4.3 Power controlled soliton switching

An important application of fibre couplers consists of using them for all optical switching. In the case of power controlled switching, an optical pulse can be switched from one output port to the other depending on its input power. To analyze switching dynamics, the coupled differential Eqs.(4.11) and (4.12) and coupled partial differential Eqs.(4.1) and (4.2) have been simulated numerically for varying dispersion parameter. The partial differential equations have been studied using finite difference beam propagation method (FDBPM) [94]. In this method an approximate solution is sought at the points of a finite grid of points and approximation of differential equation is accomplished by replacing derivatives by appropriate difference quotients. This reduces differential equation problem to a finite linear system of algebraic equations. The first step is to impose a grid on the problem and to derive the appropriate finite difference equation. Assuming an initial solution at $z = 0$ the field at the succeeding points are deduced. Let ψ_n^{k+1} denote the field at transverse grid point n and longitudinal plane k_n and assume that grid points and planes are equally spaced by Δt and Δz respectively. We impose a rectangular grid in longitudinal and radial dimension such that

$$z_k = k\Delta z, t_n = n\Delta t. \tag{4.13}$$

Finite difference equations are represented in the matrix form

$$\mathbf{A}\psi = b, \tag{4.14}$$

where the matrix **A** is tridiagonal and is solved using Thomas algorithm. When only one core, say core 1 is excited, power is transferred to the second core as pulse propagates through the core for low input power. Evolution of pulse in core 1 according to numerical simulation of partial differential equations (4.1) and (4.2) are shown in Figure 4.1. Power transfer to the second core occurs in a periodic fashion and it is plotted as a function of coupling constant for two values, as shown in Figure 4.2. The shortest distance at which maximum power is transferred to the second core for the first time is called coupling length L_c. When coupling constant increases coupling length decreases. Fraction of soliton power remaining in the arm into which it was launched is defined as transmission i.e., $T = \frac{|A_1(L_c)|^2}{|A_1(0)|^2}$. The energy transmission characteristics of a nonlinear directional coupler for periodically varying dispersion and for constant dispersion have been calculated and plotted as a function of input peak power is as shown in Figure 4.3. Critical switching power is that power for which $T = 0.5$. Critical switching power for a nonlinear directional coupler with periodically varying dispersion is 5.2, while that for a coupler with constant dispersion is 6.5. i.e. critical switching power for nonlinear directional coupler with periodically varying dispersion is lesser than that for constant dispersion.

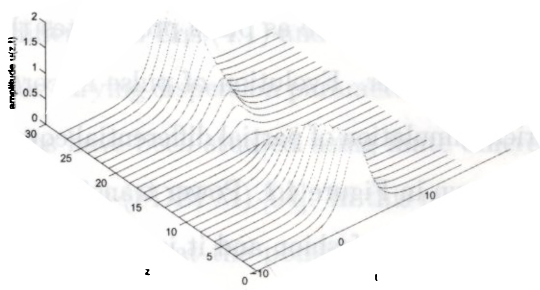

Figure 4.1: Evolution of pulse in core1 in a NLDC with varying dispersion $d(z) = cos\Omega z$ when an input pulse is launched in one core with $A_1(0) = 1.6$

4.4 Conclusion

Soliton switching in an asymmetric coupler with varying dispersion has been analyzed in this work. The coupled nonlinear Schrodinger equations which describe the pulse coupling in nonlinear directional coupler has been studied analytically and numerically. Variational analysis has been used to derive a set of coupled differential equations which describe switching dynamics. The coupled differential equations have been studied numerically for periodically varying dispersion. Finite difference beam propagation method has been used for direct partial differential equation simulation. The study has shown that for low input powers, pulse switches from one core to other and when

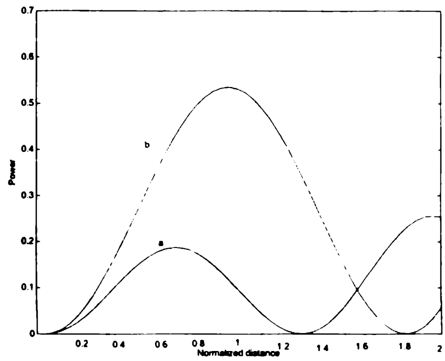

Figure 4.2: Power transferred to the second core with distance plotted as a function of coupling constant for two values. a: k=1.6, b: k=1.

input power increases pulse remains in the first core. The energy transmission characteristics of a nonlinear directional coupler for periodically varying dispersion and for constant dispersion have been calculated and plotted as a function of input peak power. The critical switching power for nonlinear directional coupler with periodically varying dispersion is lesser than that for constant dispersion. The idea of all-optical switching by taking advantages of nonlinear optical effects has fascinated the nonlinear optics community. Spatial and spatiotemporal solitons have been suggested to be ideal candidates for a number of potential practical applications including all-optical soliton steering and switching. A challenging issue is the search for physically relevant models in which stable higher dimensional spatial

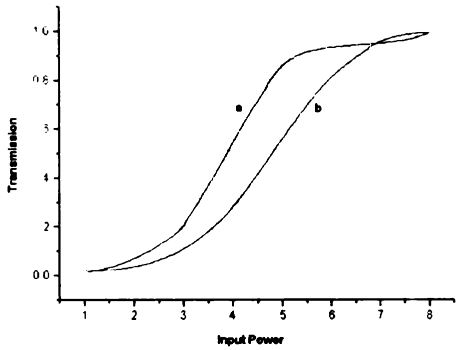

Figure 4.3: Transmission characteristics of a NLDC. a:For periodically varying dispersion $d(z) = cos\Omega z$, b:For constant dispersion.

solitons exist. Forthcoming chapters are devoted to the study of stabilization of two- dimensional spatial solitons and three-dimensional spatiotemporal solitons which exhibits a wealth of opportunities for all-optical control of light.

Chapter 5

Stabilization of two dimensional spatial solitons

5.1 Introduction

Spatial solitons in nonlinear media have attracted considerable attention in recent years. The idea of all-optical switching by taking advantages of nonlinear optical effects is a topic of interest and it is in this scenario, the self guided beams called spatial solitons find importance. The control of one light beam by another could be useful for optical communication and optical information processing. In a Kerr medium, which possesses a positive intensity dependent change of refractive index, the index increases with the light intensity. A beam of light can form a dielectric waveguide for itself, in which the refractive index is greater at the center of the beam than at its wings. The light beam in this self-formed dielectric wave guide propagates without spreading. This phenomenon can be considered as a dynamic balancing of two opposing tendencies, namely the tendency for the beam to spread due to diffraction, and the tendency for the beam to contract due to self-focussing [95]. This kind of dynamic balancing in $(1+1)$ dimensional spatial solitons are well studied, their two dimensional counterparts are less explored. Contrary to $(1+1)$ dimensional spatial solitons, a $(2+1)$ dimensional soliton that is self guided in both transverse dimension is unstable against collapse. $(2+1)$ dimensional spatial solitons in media with the Kerr nonlinearity are unstable, because two-dimensional fluctuations may destroy the balance between the nonlinearity and diffraction. In particular, an increase of the intensity leads to self-focusing of the cylindrical beam, which further increases the intensity and the corresponding intensity-dependent correction to the index of refraction, which leads to still stronger focusing and increase of the intensity, and so on. This self-accelerating process of the nonlinear self-focusing is referred to as collapse of the beam [96].

The periodic reversal of the sign of local group-velocity dispersion (GVD) known as dispersion management is a common setting in fibre optics. Dispersion managed solitons are robust to the Gordon-Haus timing jitter which makes them favorable against the ordinary solitons [97]. The strong modulation of dispersion makes it possible to achieve a high bit rate in long optical communication lines because it allows us to approach the zero dispersion limit where optical pulses do not interact strongly with one another [98]. A model governing the propagation of optical beam in a diffraction managed nonlinear waveguide array similar to dispersion management was developed [99]. Even though optical diffraction and chromatic dispersion originate from different physics, discrete diffraction spatial solitons and dispersion managed solitons share many properties which highlight the universality and diversity of solitons. Analytical and numerical treatment for the existence of dispersion managed soliton in two dimensional cubic media has been explored [100]. It has been demonstrated that the nonlinearity management can prevent the collapse of solitons in two-dimensional ($2D$) Kerr-type optical media, as well as in $2D$ Bose-Einstein condensates [101, 102, 103]. The stability of the spinning solitons also have been predicted as a result of competition between the self focusing and self de-focusing nonlinearities [104, 105, 106].

The nonlinear Schrodinger equation which governs the dynamics of two dimensional spatial solitons in Kerr media with periodically varying diffraction and nonlinearity has been analyzed in this chapter using variational approach and numerical studies. Analytical expressions for soliton parameters have been derived using variational analysis. Stability of (2 + 1) dimensional spatial soliton also has been studied using Kapitsa averaging method. Variational equations and partial differential equation have been simulated numerically. Analytical and numerical studies have shown that nonlinearity management and diffraction management stabilize the pulse against decay or collapse providing undisturbed propagation even for larger energies of the incident beam.

5.2 Wave equation with varying diffraction and nonlinearity

The field dynamics in bulk Kerr medium with varying diffraction and nonlinearity is governed by cubic nonlinear Schrodinger equation,

$$i\frac{\partial\psi}{\partial z} + d(z)\Delta\psi + \lambda(z)|\psi|^2\psi = 0, \qquad (5.1)$$

where $\lambda(z)=\lambda_0+ \lambda_1(z)$ and $d(z)=d_0+d_1(z)$ represent varying nonlinearity and varying diffraction respectively and

$\Delta=\partial^2/\partial r^2+(1/r)(\partial/\partial r)$, for axially symmetric case. The above Eq.(5.1) represents beam propagation in nonlinear bulk medium with layered structure [100]. The variational approach applied to Eq.(5.1) was developed in nonlinear optics for one dimensional ($1D$) [107] problems and then for multidimensional problems [108]. The Lagrangian density generating Eq.(5.1) is

$$\begin{aligned} L(\psi) &= \frac{ir^{D-1}}{2}\left(\psi\frac{\partial\psi^*}{\partial z}-\psi^*\frac{\partial\psi}{\partial z}\right)-d(z)r^{D-1}\left|\frac{\partial\psi}{\partial r}\right|^2 \\ &+\frac{1}{2}r^{D-1}\lambda(z)|\psi|^4. \end{aligned} \tag{5.2}$$

The asterisk stands for complex conjugation and D is the spatial dimension and $D = 2$ for two dimensions. The variational ansatz for the wave function is chosen as Gaussian:

$$\psi(r,z) = A(z)\exp\left(-\frac{r^2}{2a(z)^2}+i\frac{b(z)r^2}{2}+i\phi(z)\right). \tag{5.3}$$

where $A(z)$ is the amplitude, $a(z)$ is the beam width, $b(z)$ is the spatial chirp, and $\phi(z)$ is the phase of the wave respectively. Following the standard procedure, we insert the trial function in to the expression for Lagrangian density and calculate the effective Lagrangian as

$$L_{eff} = C_D\int_0^\infty L(\psi)dr. \tag{5.4}$$

where C_D=2π in two dimensional case.

$$L_{eff} = -\pi A^2 a^2 \frac{\partial \phi}{\partial z} - d(z) A^2 a^4 \left(\frac{1}{a^4} + b^2\right)\pi$$
$$-\frac{1}{2}\pi A^2 a^4 b_z + \frac{1}{4}\lambda(z) A^4 a^2 \pi. \qquad (5.5)$$

Varying Eq.(5.5) with respect to ϕ:

$$\frac{\delta L_{eff}}{\delta \phi} = 0 \Rightarrow \frac{\partial L_{eff}}{\partial \phi} - \frac{d}{dz}\left(\frac{\partial L_{eff}}{\partial\left(\frac{d\phi}{dz}\right)}\right) = 0, \qquad (5.6)$$

yields

$$\pi A^2 a^2 = N, \qquad (5.7)$$

where N is the conserved quantity associated with the energy of the beam. The energy of the beam is

$$E = \int_0^\infty |\psi|^2 r dr = \frac{N}{2\pi}. \qquad (5.8)$$

Now,

$$\frac{L_{eff}}{N} = -\frac{\partial \phi}{\partial z} - \frac{1}{2} a^2 b_z - d(z) a^2 \left(\frac{1}{a^4} + b^2\right) + \frac{1}{4}\lambda(z)\frac{N}{\pi a^2}. \qquad (5.9)$$

Varying (5.9) with respect to $a(z)$ and $b(z)$ yields the following equations,

$$\frac{da}{dz} = 2abd(z), \qquad (5.10)$$

and

$$\frac{db}{dz} = 2d(z)((\frac{1}{a^4}) - b^2) - \frac{N\lambda(z)}{2\pi a^4}. \qquad (5.11)$$

The equations (5.10) and (5.11) are expressions for beam width and chirp respectively.

A closed-form evolution equation for width is

$$\frac{d^2a}{dz^2} = \frac{4d(z)^2}{a^3} - \frac{d(z)N\lambda(z)}{\pi a^3} + \frac{da}{dz}\frac{d(d(z))}{dz}\frac{1}{d(z)}. \qquad (5.12)$$

Rearranging Eq.(5.12) we get

$$\frac{d}{dz}\left(\frac{1}{d(z)}\frac{da}{dz}\right) = -\frac{\partial U}{\partial a}, \qquad (5.13)$$

where U is given by

$$U = \frac{2d(z)}{a^2} - \frac{N\lambda(z)}{2\pi a^2} \qquad (5.14)$$

Hamiltonian $H(a, \frac{da}{dz}, z)$ is given as

$$H(a, \frac{da}{dz}, z) = \frac{1}{d(z)}\frac{1}{2}(\frac{da}{dz})^2 + U(a, z). \qquad (5.15)$$

The evolution of beam can be considered as the motion of a particle of variable mass $\frac{1}{d(z)}$ in a non stationary effective anharmonic potential $U(a, z)$ [109, 110]. When coefficient of diffraction and nonlinearity are constants, total energy is conserved and is given by

$$H = \frac{1}{2}(\frac{da}{dz})^2 + \frac{C}{a^2}. \tag{5.16}$$

where $C = 2d_0 - N\lambda_0/2\pi$. Obviously total energy goes to ∞ as beam width tends to zero which means that $2D$ soliton is expected to collapse for constant diffraction and nonlinearity. The condition, $C = 0$ gives the upper bound of energy, known as critical energy $E_{cr} = 2$, (when $d_0 = 1$ and $\lambda_0 = 1$) above which collapse occurs. Small fluctuations in the intensity of the incident beam, causes the intensity of the beam in the medium infinitely large, and this will finally result in the size of the beam fully diminished [111]. When diffraction and nonlinearity are periodically varying functions of the form $d(z){=}\lambda(z){=}d_0{+}d_1 \sin\Omega(z)$, Eq.(5.12) can be treated analytically by means of Kapitsa averaging method [112]. We consider the dynamics of soliton beam by separating into two parts: a rapidly oscillating part with small amplitude and a slow, smooth varying part [103]. The width of the soliton is then represented as $a(z) = a_0(z) + \rho(z)$, where $a_0(z)$ is the slowly varying part and $\rho(z)$ is the rapidly varying part with a zero mean value. Substituting $a(z)$ in Eq.(5.12) and separating the resulting equation into rapidly varying and slowly varying parts:

$$\frac{d^2\rho}{dz^2} = 2d_1 \sin\Omega(z)(2 - N/2\pi)/a_0^3, \tag{5.17}$$

$$\frac{d^2a_0}{dz^2} = \frac{2d_0(2 - N/2\pi)}{a_0^3} - \frac{6d_1(2 - N/2\pi)\overline{\sin\Omega(z)\rho(z)}}{a_0^4}, \tag{5.18}$$

where the overline indicates the average value. Integrating Eq.(5.17) twice yields

$$\rho(z) = -\frac{2d_1 \sin\Omega(z)(2 - N/2\pi)}{\Omega^2 a_0^3}. \tag{5.19}$$

Substituting Eq.(5.19) in Eq.(5.18) we obtain the equation of motion for the slowly varying part as

$$\frac{d^2a_0}{dz^2} = \frac{2d_0(2 - N/2\pi)}{a_0^3} + \frac{6d_1^2(2 - N/2\pi)^2}{\Omega^2 a_0^7}, \tag{5.20}$$

i.e,

$$\frac{d^2a_0}{dz^2} = -\frac{\partial}{\partial a_0}\left(\frac{d_0(2 - N/2\pi)}{a_0^2} + \frac{d_1^2(2 - N/2\pi)^2}{a_0^6\Omega^2}\right). \tag{5.21}$$

Hence the effective potential U for the system is given by

$$U = \frac{d_0(2 - N/2\pi)}{a_0^2} + \frac{d_1^2(2 - N/2\pi)^2}{a_0^6\Omega^2}. \tag{5.22}$$

When $N/2\pi = E_{cr} > 2$, the mechanism of collapse suppression originates from the repulsive potential near the small values of width $\sim 1/a^6$, which counteracts the attractive term $\sim 1/a^2$. The exact balance between these forces gives rise to a stable state. The plot of potential function for $N/2\pi > 2$, is as shown in Figure 5.1. When $N/2\pi > 2$, a potential well has been generated into a single point. The repulsive potential is exactly compensated by the attractive potential and stable solitons are formed. The minimum of the effective potential U is obtained at

$$a_0 = \{\frac{-3d_1^2(2 - N/2\pi)}{d_0\Omega^2}\}^{1/4}, \tag{5.23}$$

i.e. there exists one solution, with a stationary width for energy $N/2\pi > 2$ with $d_0 > 0$. Thus, periodically varying diffraction

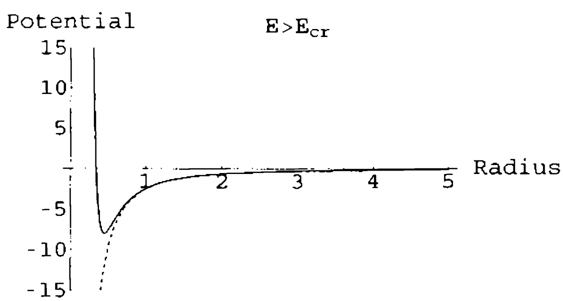

Figure 5.1: Plot of potential function U for d_0=1, d_1=3.5, Ω=50, E=$N/2\pi$=1.5. (Solid line is the plot of Eq.(5.22) and dotted line shows potential for constant diffraction and nonlinearity, i.e.d_1=0)

and nonlinearity can together stabilize two dimensional spatial solitons against collapse even for higher incident energies.

5.3 Numerical results

The differential equations (5.10) - (5.12) and partial differential Eq.(5.1) have been studied numerically for various cases of parameters of diffraction and nonlinearity. In the case of direct simulation of partial differential equation (5.1), the initial state was taken as per the ansatz (5.3), with zero chirp and the results are displayed in Figures 5.2-5.11. We consider the following cases.

Case(1): When coefficient of diffraction and nonlinearity are constants, i.e., $d_1 = 0$ and $\lambda_1 = 0$, the velocity dependent term in Eq.(5.12) vanishes. The pulse collapses when energy increases above the critical value. Variation of pulse width $a(z)$ is as shown in Figure 5.2. After a finite propagation distance, a goes to zero and chirp goes to ∞ i.e. the 2D soliton is expected to collapse. Collapse (blow-up, or self-focussing singularity) of the beam is the phenomenon where the field amplitude increases to infinity and the width of the beam decreases to zero after a finite propagation distance and this beam collapse is evident in Figure 5.3.

Case(2): When both coefficient of diffraction and nonlin-

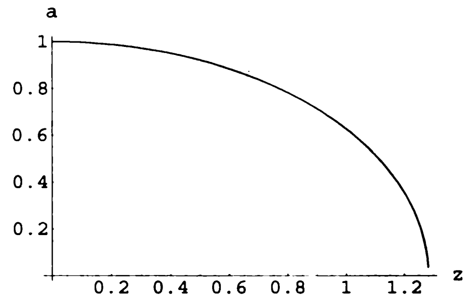

Figure 5.2: Variation of $a(z)$ for constant diffraction and nonlinearity with parameters $d_0 = 1$, $d_1 = 0$, $\lambda_0 = 1$, $\lambda_1 = 0$, a(0)=1, b(0)=0, $E = N/2\pi = 2.303$

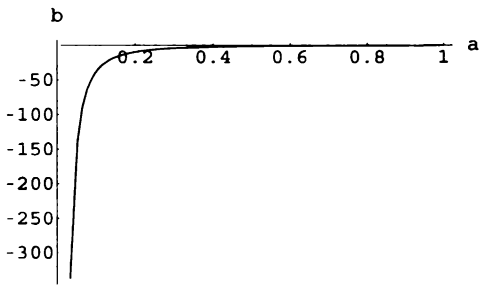

Figure 5.3: Phase portrait of variational equations for constant diffraction and nonlinearity for the same parameters as in figure5.2

earity are varying periodically, i.e. $d(z)=d_0+d_1 \sin \Omega(z)$, and $\lambda(z)=\lambda_0+\lambda_1 \sin \Omega(z)$, variation of beam width $a(z)$ with propagation distance is as shown in Figure5.4. When diffraction and nonlinearity parameters are periodically varying functions, beam width does not decrease below a critical value. This state is oscillatory. The phase portrait of variational equations (5.10) and (5.11) are as shown in Figure 5.5. Due to the periodic term in the diffraction and nonlinearity. the problem is similar to that of an inverted pendulum with an oscillating pivot point [112]. In the inverted pendulum, interplay between micro motion of the bob and the force gradient (stronger the oscillating force, the larger the deviation from the equilibrium position) produces a pseudo potential. Since pseudo potential is proportional to the square of amplitude of oscillating force, a potential barrier is formed around equilibrium position there by preventing the pendulum from swinging down [103]. Such a mechanism also stabilizes an optical beam propagating in a nonlinear medium with an alternating nonlinearity [113]. In the present case the oscillating force due to the diffraction management counteracts the self focusing and de-focusing forces induced by alternating nonlinearity. The exact balance between these forces prevent the system from collapsing. The numerical simulations of partial differential Eq.(5.1) are performed using $2D$ fast Fourier transform [114, 94]. We have considered the problem in cartesian coordinates $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$ and $r^2 = x^2 + y^2$, to perform numerical simulation. Two dimensional spatial soliton propagation according to the numerical solution of Eq.(5.1) is shown in Figure 5.6. The diffraction management and nonlinearity management stabilize the beam against decay or collapse providing undisturbed propagation even for larger energies of the incident beam.

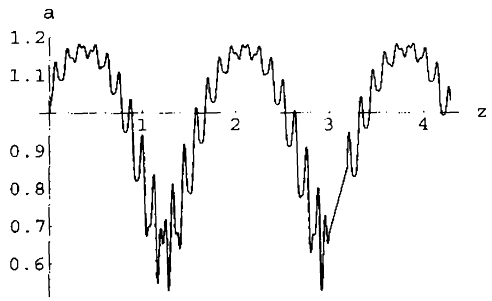

Figure 5.4: Variation of $a(z)$ for periodically varying diffraction and nonlinearity with propagation distance for the parameters $d_0 = 1$, $d_1 = 3.5$, $\lambda_0 = 1$, $\lambda_1 = -3.5$, $\Omega = 50$, $N/2\pi = 4.5$, a(0)=1, b(0)=0.

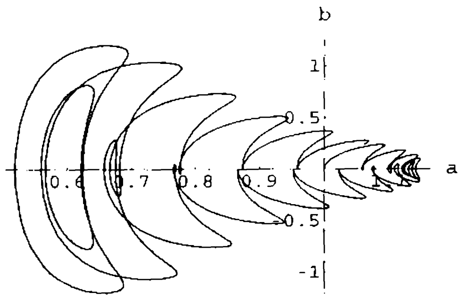

Figure 5.5: Phase portrait of variational equations for periodically varying diffraction and nonlinearity for the same parameters as in Figure 5.4

Case(3): When diffraction parameter is varying periodically and nonlinearity coefficient is a constant, i.e, $d(z)=d_0+d_1 \sin\Omega(z)$, and $\lambda(z)=\lambda_0$, stable solitons are formed for certain range of energies of the incident beam above the

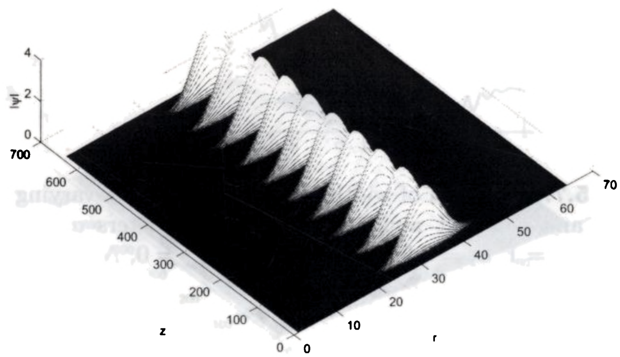

Figure 5.6: Evolution of two dimensional spatial soliton for periodically varying diffraction and nonlinearity according to numerical solution of equation (5.1) with parameters $d_0 = 1$, $d_1 = 3.5$, $\lambda_0 = 1$, $\lambda_1 = -3.5$, $\Omega = 50$ $a(0) = 1$, $b(0) = 0$, $E = N/2\pi = 4.5$.

critical value. The evolution of beam width is same as above when the energy is below 4.5. But when energy of the incident beam is increased further, the beam decays. Width evolution for an energy value 4.5 is as shown in Figure 5.7. After a finite propagation distance, beam width increases, which results in the spreading of the beam. Figure 5.8 shows the result of numerical simulation of partial differential Eq.(5.1) for periodically varying diffraction and constant nonlinearity for higher energies of the incident beam. When the beam propagates, the amplitude decreases and the beam decays. Case(4): When nonlinearity

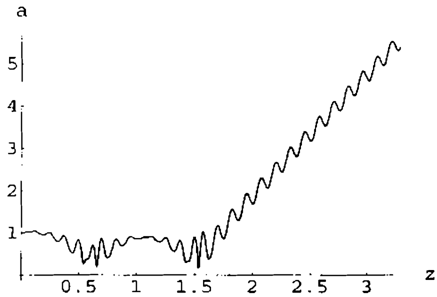

Figure 5.7: Variation of width a(z) for periodically varying diffraction and constant nonlinearity with parameters $a(0) = 1$, $b(0) = 0$, $d_0 = 1$, $d_1 = 3.5$, $\Omega = 50$, $\lambda_0 = 1$, $\lambda_1 = 0$, $N/2\pi = 4.5$

is varying periodically and diffraction coefficient is a constant, i.e, $d(z)=d_0$, $\lambda(z)=\lambda_0+\lambda_1 \sin\Omega(z)$, the soliton is stable for certain range of energies of the incident beam above critical value. The phase portrait of variational equations for energy=2.303 is shown in Figure 5.9. On further increase of energy, the beam collapses after a finite propagation distance. Variation of beam width a(z) for constant diffraction and periodically varying nonlinearity is as shown in Figure 5.10. The beam width decreases and tends to zero after a finite propagation distance i.e, amplitude increases and this self-focusing leads to the collapse of the beam. Figure 5.11 is the numerical solution of Eq.(5.1) which shows the self-focusing of the beam.

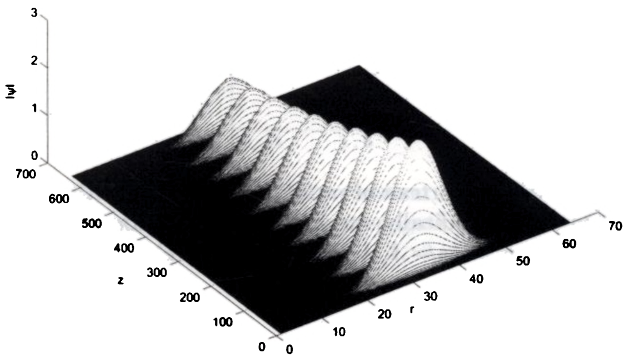

Figure 5.8: Decay of pulse amplitude of two dimensional diffraction managed soliton with a(0)=1, b(0)=0, $d_0 = 1$, $d_1 = 3.5$, $\lambda_0 = 1$, $\lambda_1 = 0$, $\Omega = 50$ $E = N/2\pi$=4.5.

5.4 Conclusion

We have explored the existence of two dimensional spatial soliton in Kerr-media with periodically varying diffraction and nonlinearity. Variational approach has been used to derive a set of ordinary differential equations which describe the optical beam evolution. Analytical and numerical studies have shown that the periodic force arising due to the periodically varying diffraction and nonlinearity stabilize the two dimensional spatial solitons in Kerr media. Kapitsa averaging method has been used to analyze the form of potential function. For greater incident

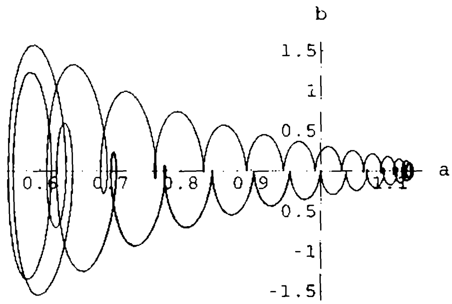

Figure 5.9: Phase portrait of variational equations with constant diffraction and periodically varying nonlinearity for the parameters $d_0 = 1$, $d_1 = 0$, $\lambda_0 = 1$, $\lambda_1 = 3.5$, $\Omega = 50$, $E = N/2\pi = 2.303$

energies, when the coefficient of diffraction and nonlinearity are periodically varying functions, a potential well has been generated into a single point which means that the stable solitons are formed. Thus, periodically varying diffraction and nonlinearity can together stabilize two dimensional spatial solitons against collapse even for higher incident energies. When the coefficient of diffraction and nonlinearity are constants, the beam collapses after propagating through a finite distance. When diffraction is managed keeping nonlinearity constant, stable solitons are formed for lower energy of the incident beam, but for larger energies, the beam decays. When nonlinearity is managed, keeping diffraction constant, then the beam collapses for higher incident energies. Analytical and numerical studies have shown that the diffraction management and nonlinearity management can sta-

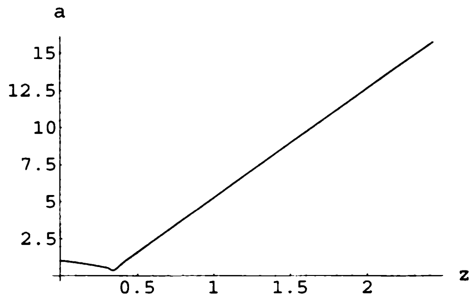

Figure 5.10: Variation of $a(z)$ for constant diffraction and periodically varying nonlinearity with propagation distance for the parameters $d_0 = 1$, $d_1 = 0$, $\lambda_0 = 1$, $\lambda_1 = 3.5$, $\Omega = 50$, $E = N/2\pi = 4.5$

bilize the beam against decay or collapse providing undisturbed propagation even for larger energies of the incident beam. The forthcoming chapter deals with study of a confined wave packet in (2+1+1) dimension which represents the extension of a self-trapped optical beam into the temporal domain and is known as Spatiotemporal soliton.

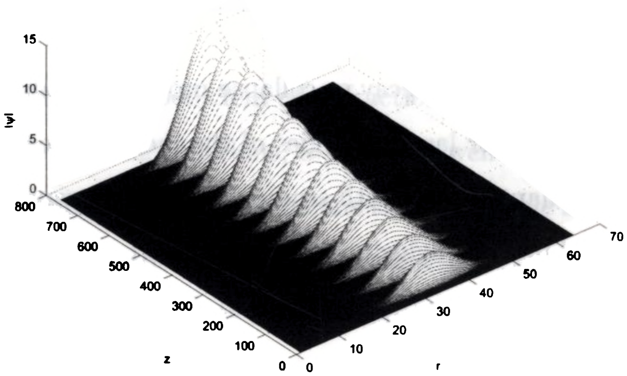

Figure 5.11: Self-focusing of two dimensional spatial soliton for constant diffraction and periodically varying nonlinearity with parameters $d_0 = 1$, $d_1 = 0$, $\lambda_0 = 1$, $\lambda_1 = 3.5$, $\Omega = 50$, $a(0) = 1$, $b(0) = 0$, $E = N/2\pi$=4.5

Chapter 6

Stable Light bullets in Kerr media

6.1 Introduction

One of the major goals in the field of soliton physics is the generation of pulses that are localized in all the transverse dimensions of space, as well as in time. These are (2+1+1) dimensional objects referred to as spatiotemporal solitons (STS), where 2 refers to the transverse dimensions, the first 1 stands for the temporal variable, and the last 1 pertains to the propagation coordinate [115]. Spatio-temporal soliton results from the simultaneous balance of diffraction and group velocity dispersion (GVD) by the transverse self-focusing and nonlinear phase modulation in the longitudinal direction respectively. The possibility of such pulses in Kerr media were considered by Silberberg [116], who coined the term light bullets (LB) for them, which stresses their particle like nature. In a Kerr medium the light bullets in higher dimensions are unstable against spatio-temporal collapse induced by the combined effects of diffraction, anomalous dispersion and nonlinearity. Though they are unstable in the uniform self-focusing Kerr medium [117] stability can be achieved in saturable [118, 119, 120] quadratically nonlinear [121, 122, 123] graded-index Kerr media [124]. Three dimensional spatio-temporal solitons in a bulk medium have not yet been observed in an experiment. The scientific importance of STS has motivated a number of studies on STS and were predicted in many works [125, 126, 127]. The study of stabilization of three-dimensional spatiotemporal solitons is a field of growing interest. The challenge in the quest for spatiotemporal solitons, or light bullets, is to identify physically relevant models of nonlinear optical systems, based on evolution equations that allow stable three-dimensional propagation. Actually, the dimensionality is a central issue in the formation of solitons because of the problem of stability against the collapse in the multidimensional case. By using the diffraction properties of waveguide arrays, a scheme to produce structures with designed diffraction was discussed [128]. The existence of stable dispersion managed two dimensional solitons in cubic media has been proposed [129]. Recently, it was suggested that a rapid variation of nonlinear Kerr coefficient can also stabilize (2+1) dimensional solitons [130, 131].

In this work, the existence of stable three dimensional spatiotemporal solitons in bulk Kerr media with periodically varying dispersion, diffraction and nonlinearity has been explored, by means of variational approximation and Kapitsa averaging method.

6.2 Mathematical model

The time-dependent nonlinear Schrodinger equation (NLSE) representing wave motion in a cubic medium with varying dispersion, diffraction and nonlinearity is given by

$$i\frac{\partial\psi}{\partial z} + d(z)\nabla_T^2\psi + D(z)\frac{\partial^2\psi}{\partial\tau^2} + \lambda(z)|\psi|^2\psi = 0, \qquad (6.1)$$

where the diffraction operator ∇_T^2 acts on transverse coordinates x and y, τ is the reduced temporal variable and the propagation of the pulsed beam is along the 'z' direction. The parameters $d(z)$, $D(z)$ and $\lambda(z)$ are the coefficients of diffraction.

dispersion and nonlinearity respectively. The above functions can be simple harmonic functions of the form, $D(z)$ $=d(z)=$ $d_0+d_1\sin(\Omega z)$ and $\lambda(z)=\lambda_0+$ $\lambda_1\sin(\Phi z)$. If we introduce radial variable $r=(x^2+y^2)^{\frac{1}{2}}$, Eq.(6.1) can be written as

$$i\frac{\partial\psi}{\partial z}+d(z)(\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial\psi}{\partial r}))+d(z)\frac{\partial^2\psi}{\partial\tau^2}+\lambda(z)|\psi|^2\psi=0, \qquad (6.2)$$

where $\nabla_T^2=\partial^2/\partial r^2+(1/r(\partial/\partial r))$ for axially symmetric case.

We follow a variational technique to describe the beam evolution based on Lagrangian formalism of classical mechanics, proposed and developed in nonlinear optics for one dimensional ($1D$) problems and then for multidimensional problems.

The Lagrangian density of Eq.(6.2) is

$$L(\psi)=\frac{ir^2}{2}\left(\frac{\partial\psi}{\partial z}\psi^*-\psi\frac{\partial\psi^*}{\partial z}\right)-d(z)r^2|\frac{\partial\psi}{\partial r}|^2$$
$$-D(z)r^2|\frac{\partial\psi}{\partial\tau}|^2+\frac{1}{2}r^2\lambda(z)|\psi|^4 \qquad (6.3)$$

Following the Ritz optimization procedure let us assume the initial profile as Gaussian

$$\psi(r,\tau,z)=A(z)\exp\left(-\frac{r^2}{2a(z)^2}+i\frac{b(z)r^2}{2}-\frac{\tau^2}{2T(z)^2}+i\frac{\beta(z)\tau^2}{2}+i\phi(z)\right), \qquad (6.4)$$

where $A(z)$, $\phi(z)$, $a(z)$, $T(z)$, $b(z)$ and $\beta(z)$ are the amplitude phase, spatial width, temporal width, spatial chirp and temporal chirp respectively. Following the standard procedure, we insert the trial function into the expression for Lagrangian density and calculate the effective Lagrangian as

$$L_{eff} = C_D \int_0^{\infty} L(\psi) dr d\tau, \tag{6.5}$$

where C_D=4π in three dimensional case. The effective Lagrangian [108]is obtained as

$$L_{eff} = -\pi^{\frac{3}{2}} A^2 a^2 T(\phi_z - \frac{1}{2} a^2 b_z - \frac{1}{4} T^2 \beta_z$$
$$-d(z)(\frac{1}{a^2} + b^2 a^2) - \frac{1}{2} d(z)(\frac{1}{T^2} + T^2 \beta^2) + \frac{1}{4\sqrt{2}} \lambda(z) A^2). \tag{6.6}$$

The variational equation,

$$\frac{\delta L_{eff}}{\delta \phi} = 0 \Rightarrow \frac{\partial L_{eff}}{\partial \phi} - \frac{d}{dz} \left(\frac{\partial L_{eff}}{\partial \left(\frac{d\phi}{dz} \right)} \right) = 0, \tag{6.7}$$

yields the conservation relation,

$$\pi^{\frac{3}{2}} A^2 a^2 T = N, \tag{6.8}$$

where N is the conserved quantity associated with the energy of the beam. The energy of the beam is

$$E = \int_0^\infty |\psi|^2 r^2 dr d\tau \tag{6.9}$$

Now,

$$\begin{aligned} L_{eff}/N = & -\phi_z - \frac{1}{2}a^2 b_z - \frac{1}{a^2}d(z) - d(z)a^2b^2 + \frac{1}{4}T^2\beta_z \\ & -\frac{1}{2T^2}D(z) - D(z)T^2\beta^2 + \frac{\lambda(z)N}{4a^2T\pi^{\frac{3}{2}}\sqrt{2}}. \end{aligned} \tag{6.10}$$

A set of evolution equations can be derived from L_{eff}/N taking into account that the variation with respect to the unknowns in the initial profile is equal to zero. Varying Eq.(6.10) with respect to $a(z)$, $b(z)$, $T(z)$, $\beta(z)$ yield the following equations.

$$\frac{da}{dz} = 2abd(z), \tag{6.11}$$

$$\frac{db}{dz} = \frac{2d(z)}{a^4} - 2d(z)b^2 - \frac{N\lambda(z)}{2\sqrt{2}\pi^{\frac{3}{2}}a^4T}, \tag{6.12}$$

$$\frac{dT}{dz} = 2T\beta d(z), \tag{6.13}$$

$$\frac{d\beta}{dz} = \frac{2d(z)}{T^4} - 2d(z)\beta^2 - \frac{N\lambda(z)}{2\sqrt{2}\pi^{\frac{3}{2}}a^2T^3}, \tag{6.14}$$

$$\frac{d^2T}{dz^2} = \frac{4d(z)^2}{T^3} - \frac{d(z)N\lambda(z)}{\pi^{\frac{3}{2}}T^2\sqrt{2}a^2} + 2T\beta\left(\frac{d(d(z))}{dz}\right), \tag{6.15}$$

$$\frac{d^2a}{dz^2} = \frac{4d(z)^2}{a^3} - \frac{d(z)N\lambda(z)}{\pi^{\frac{3}{2}}a^3T\sqrt{2}} + 2ab(\frac{d(d(z))}{dz}). \quad (6.16)$$

If we assume $a = T$ and rearranging Eq.(6.16) we obtain,

$$\frac{d}{dz}\left(\frac{1}{d(z)}\frac{da}{dz}\right) = -\frac{\partial U}{\partial a}. \quad (6.17)$$

where U is given by

$$U = \frac{2d(z)}{a^2} - \frac{N\lambda(z)}{3\sqrt{2}\pi^{3/2}a^3} \quad (6.18)$$

Hamiltonian $H(a, \frac{da}{dz}, z)$ is given as

$$H(a, \frac{da}{dz}, z) = \frac{1}{d(z)}\frac{1}{2}(\frac{da}{dz})^2 + U(a, z). \quad (6.19)$$

The evolution of the beam can be considered as motion of a particle of variable mass $\frac{1}{d(z)}$ in non stationary effective anharmonic potential $U(a, z)$.

$$H(a, \frac{da}{dz}, z) = \frac{1}{2d_0}(\frac{da}{dz})^2 + \frac{2d_0}{a^2} - \frac{N\lambda_0}{3a^3\sqrt{2}\pi^{3/2}}. \quad (6.20)$$

where $d(z) = d_0$, $\lambda(z)$=λ_0. It is obvious that the spatiotemporal soliton energy goes to infinity as the beam width goes to zero. In other words, the intensity becomes infinitely large at a finite distance as the size of the beam diminishes and tends to zero both in spatial and temporal domains. The potential well description for the variation of beam width conveys a physical picture of the competition between between the repulsive and attractive effects.

The plot of potential function for constant parameters is as shown in Figure 6.1. The attractive potential increases with decreasing width and the beam collapses for constant coefficients of diffraction, dispersion and nonlinearity.

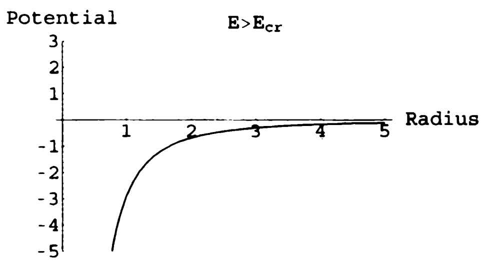

Figure 6.1: Variation of U with for constant dispersion, diffraction and nonlinearity

6.3 Averaged Variational equations

When coefficients of diffraction, dispersion and nonlinearity are periodically varying functions, the dynamics of soliton can be treated analytically on two scales rapid oscillations over one period and accumulation of small deviations from periodic oscillations over many periods by means of Kapitsa averaging method. We have considered Kapitsa averaging method for different cases.

Case1: When $d(z)$ is a periodically varying function of the form $d(z)= d_0+d_1 \sin\Omega(z)$ and $\lambda(z)=\lambda_0$, a constant, We consider the dynamics of soliton beam by separating into two parts: a rapidly oscillating part with small amplitude and a slow, smooth varying part [103]. The width of the soliton is then represented as $a(z) = a_0(z)+\rho(z)$, where $a_0(z)$ is the slowly varying part and $\rho(z)$ is the rapidly varying part with a zero mean value. Substituting $a(z)$ in Eq.(6.16) and separating the resulting equation into rapidly varying and slowly varying parts:

$$\frac{d^2\rho}{dz^2} = \frac{4d_1 \sin(\Omega z)}{a_0^3}, \tag{6.21}$$

and

$$\frac{d^2 a_0}{dz^2} = \frac{4d_0}{a_0^3} - \frac{N\lambda_0}{\sqrt{2}\pi^{3/2}a_0^4} - \frac{12d_1\overline{\sin(\Omega z)\rho(z)}}{a_0^4} \tag{6.22}$$

where the overline indicates the average value. Integrating Eq.(6.21) we obtain

$$\rho(z) = -\frac{4d_1 \sin(\Omega z)}{\Omega^2 a_0^3}. \tag{6.23}$$

Substituting Eq.(6.23) in Eq.(6.22) we obtain the equation of motion for the slowly varying part as

$$\frac{d^2a_0}{dz^2} = \frac{4d_0}{a_0^3} - \frac{N\lambda_0}{\sqrt{2}\pi^{3/2}a_0^4} + \frac{24d_1^2}{\Omega^2 a_0^7} \tag{6.24}$$

i.e.,

$$\frac{d^2a_0}{dz^2} = -\frac{\partial}{\partial a_0}\left(\frac{2d_0}{a_0^2} + \frac{4d_1^2}{a_0^6\Omega^2} - \frac{N\lambda_0}{3\sqrt{2}\pi^{3/2}a_0^3}\right) \tag{6.25}$$

Hence the effective potential U for the system is given by

$$U = \frac{2d_0}{a_0^2} + \frac{4d_1^2}{a_0^6\Omega^2} - \frac{N\lambda_0}{3\sqrt{2}\pi^{3/2}a_0^3}. \tag{6.26}$$

The dotted line in Figure 6.2 shows the form of potential for periodically varying $d(z)$ and constant $\lambda(z)$. The repulsive potential increases for smaller widths and hence the beam decays for smaller widths. Case2: When $\lambda(z)$ is a periodically varying function of the form $\lambda(z) = \lambda_0 + \lambda_1 \sin(\Omega z)$ and $d(z) = d_0$, we get the following equations.

$$\frac{d^2\rho}{dz^2} = -\frac{N\lambda_1 \sin(\Omega z)}{\sqrt{2}\pi^{3/2}a_0^4}, \tag{6.27}$$

and

$$\frac{d^2a_0}{dz^2} = \frac{4d_0}{a_0^3} - \frac{N\lambda_0}{\sqrt{2}\pi^{3/2}a_0^4} + \frac{N^2\lambda_1^2}{\pi^3\Omega^2 a_0^9}, \tag{6.28}$$

where

$$\rho(z) = \frac{N\lambda_1 \sin(\Omega z)}{\Omega^2\sqrt{2}\pi^{3/2}a_0^4}. \tag{6.29}$$

Hence the effective potential U for the system is given by

$$U = \frac{2d_0}{a_0^2} - \frac{N\lambda_0}{3\sqrt{2}\pi^{3/2}a_0^3} + \frac{N^2\lambda_1^2}{8\pi^3 a_0^8\Omega^2}. \tag{6.30}$$

The dashed line in Figure 6.2 shows the form of potential for periodically varying nonlinearity and constant $d(z)$. When width decreases repulsive potential increases and beam decays.

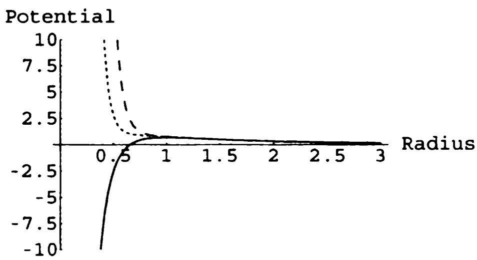

Figure 6.2: Plot of potential function for three cases. Solid line shows the plot for $d(z)= d_0$ and $\lambda(z)=\lambda_0$, dashed line shows the potential for $\lambda(z) = \lambda_0 + \lambda_1 \sin(\Omega z)$ and $d(z)= d_0$, dotted line shows the potential for $d(z)= d_0+d_1 \sin\Omega(z)$ and $\lambda(z)=\lambda(z)_0$.

Case3: When $d(z)$ and $\lambda(z)$ are periodically varying function of the form $d(z)= d_0+d_1 \sin\Omega(z)$ and $\lambda(z) =\lambda_0+\lambda_1 \sin\Omega(z)$, we get the following equations.

$$\frac{d^2\rho}{dz^2} = \frac{4d_1\sin(\Omega z)}{a_0^3} - \frac{N\lambda_1}{\sqrt{2}\pi^{3/2}a_0^4}, \tag{6.31}$$

$$\frac{d^2a_0}{dz^2} = \frac{4d_0}{a_0^3} - \frac{N\lambda_0}{\sqrt{2}\pi^{3/2}a_0^4} - \frac{12d_1\overline{\sin(\Omega z)\rho(z)}}{a_0^4} + \frac{4N\lambda_1\overline{\sin(\Omega z)\rho(z)}}{\sqrt{2}\pi^{3/2}a_0^5}, \tag{6.32}$$

where the overline indicates the average value. Integrating Eq.(6.31) twice yields

$$\rho(z) = -\frac{\sin(\Omega z)}{\Omega^2a_0^3}(4d_1 - N\lambda_1/\sqrt{2}\pi^{3/2}a_0). \tag{6.33}$$

Substituting Eq.(6.33) in Eq.(6.32) we obtain the equation of motion for the slowly varying part as

$$\frac{d^2a_0}{dz^2} = \frac{4d_0}{a_0^3} - \frac{N\lambda_0}{\sqrt{2}\pi^{3/2}a_0^4} + \frac{24d_1^2}{\Omega^2a_0^7} - \frac{12.242Nd_1\lambda_1}{\pi^{3/2}\Omega^2a_0^8} + \frac{N^2\lambda_1^2}{\pi^3\Omega^2a_0^9}, \tag{6.34}$$

there for the effective potential U for the system is given by

$$U = \frac{2d_0}{a_0^2} + \frac{4d_1^2}{a_0^6\Omega^2} - \frac{12.242Nd_1\lambda_1}{7\pi^{3/2}\Omega^2a_0^7} - \frac{N\lambda_0}{3\sqrt{2}\pi^{3/2}a_0^3} + \frac{N^2\lambda_1^2}{8\pi^3\Omega^2a_0^8}. \tag{6.35}$$

The plot of potential function Figure 6.3 shows the form of potential for periodically varying dispersion, diffraction and nonlinearity. When $d(z)$ and $\lambda(z)$ are periodically varying functions, a potential well has been generated into a single point due to the competition between the repulsive and attractive forces for smaller widths. i.e, the repulsive potential is exactly compensated by the attractive potential and stable solitons are formed. Thus, periodically varying diffraction, dispersion and nonlinearity can stabilize three dimensional light bullets against collapse or decay of the beam.

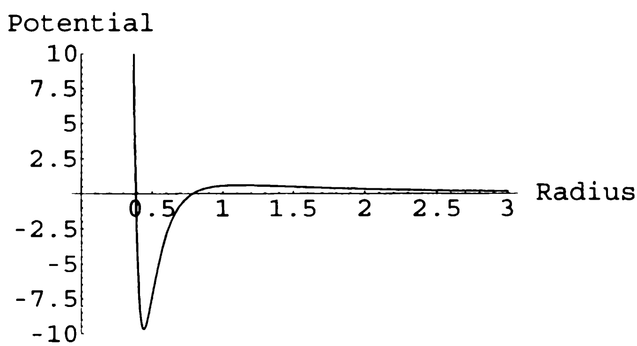

Figure 6.3: Plot of potential function for $\lambda(z) = \lambda_0 + \lambda_1 \sin(\Omega z)$ and $d(z) = d_0 + d_1 \sin \Omega(z)$

6.4 Conclusion

The stabilization of spatiotemporal solitons with periodically varying dispersion, diffraction and nonlinearity has been analyzed using variational approach and Kapitsa averaging method. The averaged equations for the potential function has been derived. When the parameters of dispersion, diffraction and nonlinearity are constants, attractive potential increases for smaller widths indicating the decay of the beam. When nonlinearity or dispersion and diffraction are periodically varying functions, repulsive potential dominates for smaller widths and hence the beam decays. When $d(z)$ and $\lambda(z)$ are periodically varying functions, a potential well has been generated into a single point due to the exact balance between repulsive and attractive potential and stable solitons are formed. i.e., light bullets get stabilized in Kerr media due to the combined effect of periodically varying coefficients of dispersion, diffraction and nonlinearity.

Chapter 7

Spatial and spatiotemporal solitons in cubic-quintic media

7.1 Introduction

Studies on solitary beams have recently been extended to near-Kerr media with competing nonlinearities [132]. The nonlinear refractive index of certain materials begins to deviate from the $n_2 I$ for large intensities. A model which has attracted considerable attention is the one with a cubic-quintic (CQ) nonlinearity. In this case, the refractive index has the form, $n = n_0 + n_2 I - n_4 I^2$, where I is the beam intensity. A unifying feature of those media is competition between self focusing cubic and self de-focusing quintic nonlinearities. CQ nonlinearity is important mainly in multidimensional case, as the combination of the self focusing cubic and self de-focusing quintic terms prevent collapse and make it possible to anticipate the existence of stable solitons [133, 134]. It has been shown that CQ nonlinearity describes the dielectric response of poly diacetylene p-toluene sulphonate (PTS) crystal and certain semiconductor doped glasses [135]. Stable spinning solitons also have been found in cubic-quintic media [136]. The importance of cubic-quintic nonlinear Schrodinger equation (NLSE), which governs the evolution of the beam intensity in a cubic-quintic medium, apart from nonlinear optics emerges in a variety of areas, like biophysics, plasma theory, nuclear hydrodynamics, etc. [137, 138]. The existence and stability of solitons in an optical wave guide equipped with a Bragg grating in which nonlinearity contains both cubic and quintic terms has been investigated [139]. Stable high-energy spinning solitons can exist in media with competing nonlinearities, such as self focusing cubic and self-de focusing quintic nonlinearities [140, 141]. Numerical and analytical investigations of beam propagation in cubic-quintic media have demonstrated the feasibility of self trapping [142, 143]. Recently, there has been a great interest in understanding the physical conditions required for the creation of stable multidimensional solitons. It is observed that even in cubic-quintic medium the beam collapses for higher incident energies. This chapter analyzes the stabilization of two dimensional spatial soliton and three dimensional spatiotemporal solitons in cubic-quintic media. A diffraction managed model in cubic-quintic media has been formulated and studied for the stabilization of spatial solitons. A dispersion managed model with diffraction management has been proposed for the stabilization of three dimensional spatiotemporal soilton in bulk cubic-quintic media. The cubic-quintic nonlinear Schrodinger equation with periodically varying dispersion and diffraction has been studied using analytically and numerically. Analytical and numerical studies have shown that diffraction management stabilizes the two dimensional spatial soliton against collapse while periodically varying coefficients of diffraction and dispersion stabilizes the spatiotemporal solitons in cubic-quintic media.

7.2 Diffraction managed cubic quintic model

The field dynamics in a cubic quintic medium is governed by NLSE [144, 145]

$$2ik\frac{\partial\psi}{\partial z}+\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+2kk_0n_2|\psi|^2\psi-2kk_0n_4|\psi|^4\psi=0, \quad (7.1)$$

where the propagation of the pulsed beam is along the z direction and $k=\omega/c$, $k_0=n_0k$, n_0 is the linear refractive index, and the positive constants n_2 and n_4 characterize the dependence of the refractive index on the intensity of the beam and ψ is amplitude of the beam. Normalizing Eq. (7.1) as $z'=\alpha z/z_0$, $x'=\sqrt{\alpha}x/r_0$, $y'=\sqrt{\alpha}y/r_0$, $\psi'^2=\psi^2/\alpha\psi_0^2$ with $z_0=n_4/2kk_0n_2^2$, $r_0=\sqrt{n_4/2kk_0n_2^2}$, $\psi_0^2=n_4/n_2$.

$$i\frac{\partial\psi}{\partial z}+\nabla_T^2\psi+|\psi|^2\psi-\alpha|\psi|^4\psi=0, \quad (7.2)$$

where the parameter α, is coefficient of quintic nonlinearity and operator ∇_T^2 acts on transverse coordinates x and y. In Eq.(7.2) primes have been omitted for simplicity. The field dynamics in a cubic-quintic medium with varying diffraction is governed by cubic-quintic NLSE of the form

$$i\frac{\partial\psi}{\partial z} + d(z)\Delta\psi + |\psi^2|\psi - \alpha|\psi|^4\psi = 0, \quad (7.3)$$

where we have introduced radial variable $r = (x^2 + y^2)^{\frac{1}{2}}$, $\Delta = \partial^2/\partial r^2 + (1/r)(\partial/\partial r)$ for axially symmetric case. The coefficient $d(z) = d_0 + d_1(z)$ represents varying diffraction. Our analysis of soliton dynamics under diffraction management is based on variational approach. According to which the Lagrangian density generating Eq.(7.3) is

$$\begin{aligned} L(\psi) &= \frac{ir}{2}\left(\psi\frac{\partial\psi^*}{\partial z} - \psi^*\frac{\partial\psi}{\partial z}\right) - d(z)r|\frac{\partial\psi}{\partial r}|^2 \\ &+ \frac{1}{2}r|\psi|^4 - \frac{\alpha}{3}r|\psi|^6. \end{aligned} \quad (7.4)$$

Let us assume a trial solution of the form:

$$\psi(r, z) = A(z)\exp\left(-\frac{r^2}{2a(z)^2} + i\frac{b(z)r^2}{2} + i\phi(z)\right), \quad (7.5)$$

where $A(z)$, $a(z)$, $b(z)$, $\phi(z)$ are the amplitude, width, chirp and phase respectively. The effective Lagrangian is given by,

$$L_{eff} = C_D\int_0^\infty L(\psi)dr, \quad (7.6)$$

where $C_D=2\pi$ in two dimensional case and we obtain the effective Lagrangian as

$$L_{eff} = -\pi A^2 a^2 \frac{\partial \phi}{\partial z} - \frac{1}{2}\pi A^2 a^4 b_z - d(z) A^2 a^4 \left(\frac{1}{a^4} + b^2\right)\pi + \frac{1}{4}A^4 a^2 \pi - \frac{\alpha}{9} A^6 a^2 \pi. \quad (7.7)$$

The equations for soliton parameters are derived from L_{eff} by taking variations of Eq.(7.7) with respect to $\phi(z)$, $a(z)$ and $b(z)$. Then we obtain

$$\pi A^2 a^2 = N, \quad (7.8)$$

$$\frac{da}{dz} = 2abd(z), \quad (7.9)$$

$$\frac{db}{dz} = 2d(z)((\frac{1}{a^4}) - b^2) - \frac{N}{2\pi a^4} + \frac{4\alpha N^2}{9\pi^2 a^6}. \quad (7.10)$$

Eqs.(7.9) and (7.10) give the expressions for beam width and chirp respectively. The space averaged Lagrangian is

$$\frac{L_{eff}}{N} = -\frac{1}{2}a^2 b_z - \phi_z - \frac{d(z)}{a^2} - d(z)a^2 b^2 + \frac{N}{4\pi a^2} - \frac{\alpha N^2}{9\pi^2 a^4} \quad (7.11)$$

A closed-form evolution equation for width is

$$\frac{d^2 a}{dz^2} = \frac{4d(z)^2}{a^3} - \frac{d(z)N}{\pi a^3} + \frac{8\alpha N^2 d(z)}{9\pi^2 a^5} + \frac{da}{dz}\frac{d(d(z))}{dz}\frac{1}{d(z)}, \quad (7.12)$$

i.e,

$$\frac{d}{dz}\left(\frac{1}{d(z)}\frac{da}{dz}\right) = -\frac{\partial U}{\partial a}. \tag{7.13}$$

where U is given by

$$U = \frac{2d(z)}{a^2} - \frac{N}{2\pi a^2} + \frac{2\alpha N^2}{9a^4\pi^2}. \tag{7.14}$$

Hamiltonian $H(a, \frac{da}{dz}, z)$ is given as

$$H(a, \frac{da}{dz}, z) = \frac{1}{d(z)}\frac{1}{2}(\frac{da}{dz})^2 + U(a, z). \tag{7.15}$$

The evolution of beam can be considered as motion of a particle of variable mass $\frac{1}{d(z)}$ described by new coordinates a and b in non stationary effective anharmonic potential $U(a, z)$. When $d(z)$ is a periodic function, that is $d(z)$=d_0+ $d_1 \sin\Omega(z)$, the equation of motion becomes

$$\frac{d^2a}{dz^2} = \frac{4(d_0 + d_1\sin\Omega(z))^2}{a^3} - \frac{(d_0 + d_1\sin\Omega(z))N}{\pi a^3} + \frac{8\alpha N^2(d_0 + d_1\sin\Omega(z))}{9\pi^2 a^5} + 2abd_1\Omega\cos\Omega(z). \tag{7.16}$$

Due to the oscillating term in the equation of motion, the system becomes oscillatory. The velocity dependent term in equation (7.12) describes the oscillatory dynamics of $2D$ diffraction managed solitons.

7.3 Numerical results and discussion

The coupled differential Eqs.(7.9) and (7.10)and partial differential Eq.(7.1) are studied numerically. When diffraction is a periodically varying function, i.e., $d(z)=d_0+d_1 \sin\Omega(z)$, the variation of beam width $a(z)$ and chirp parameter $b(z)$ with propagation distance are shown in Figures (7.1) and (7.2) respectively. The beam width and chirp become oscillatory. A net stabilizing force may be produced by attractive and repulsive forces arising from diffraction management and competing nonlinearities. The exact balance between these forces give rise to a stable state. This stabilization mechanism is similar to that of an inverted pendulum. A pendulum with an oscillating pivot can have a stable equilibrium with its bob situated above the pivot. The inverted pendulum reaches a stable configuration due to the net stabilizing force produced by stabilizing and destabilizing forces at a frequency much faster than the natural oscillation frequency of the fixed-pivot pendulum. Such a mechanism also stabilizes a Bose-Einstein condensate in a double-well potential with oscillating interaction and an optical beam propagating in a medium with alternating nonlinearity. The phase portrait of variational equations for periodically varying diffraction is as shown in Figure 7.3.

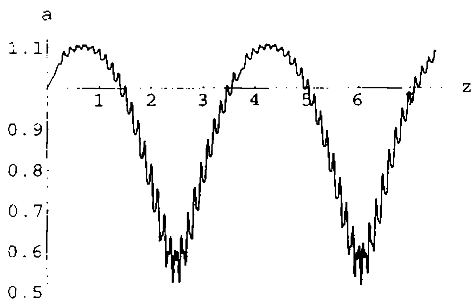

Figure 7.1: Quasi periodic dynamics of width $a(z)$ of a diffraction managed 2D soliton with cubic-quintic nonlinearity for the parameters $d_0 = 1$, $d_1 = 3.5$, $\Omega = 50$, $N = 14.717$

When diffraction is a constant, i.e., $d(z)$=d_0 the variation of beam width $a(z)$ and chirp parameter $b(z)$ with propagation distance are shown in Figures (7.4) and (7.5) respectively. After a finite propagation distance a goes to zero which is the self focusing singularity and b goes to ∞. Collapse (or blow-up, or self-focussing singularity) of the beam is the phenomenon where the field amplitude increases to infinity and the width of the beam decreases to zero after a finite propagation distance. When diffraction is a constant, $2D$ soliton collapses for higher incident energies even in cubic-quintic media.

The partial differential Eq.(7.1) that governs the dynamics of spatial solitons in cubic-quintic media is studied numerically for periodically varying diffraction coefficient using finite difference beam propagation method (FD-BPM) [61]. The initial solu-

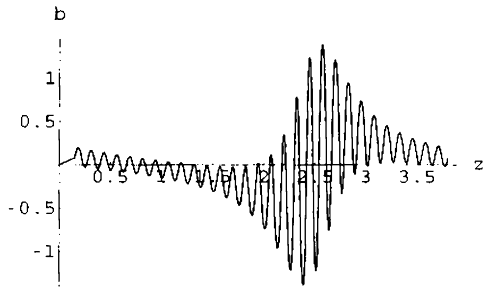

Figure 7.2: Variation of chirp $b(z)$ of a diffraction managed 2D soliton with cubic-quintic nonlinearity for the same parameters as in figure (7.1).

tion was chosen as per ansatz given in Eq. (7.5).The numerical parameters of simulation has been chosen so as to fit the experimental configurations. The nonlinear parameters chosen are $n_0 = 3.3$, $n_2 = 1.5 \times 10^{-13} cm^2/W$, $n_4 = -5 \times 10^{-23} cm^4/W^2$ at $1.55\mu m$ wavelength and $n_0 = 1.6755$, $n_2 = 2.2 \times 10^{-12} cm^2/W$, $n_4 = -8 \times 10^{-22} cm^4/W^2$ at $1.6\mu m$ [150]. The parameters of diffraction are $d_0 = 1$, $d_1 = 3.5$, $\Omega = 50$. The evolution of diffraction managed soliton in cubic-quintic media according to numerical simulation shown in Figure 7.6 reveals that the diffraction management stabilizes the beam against decay, providing undisturbed propagation.

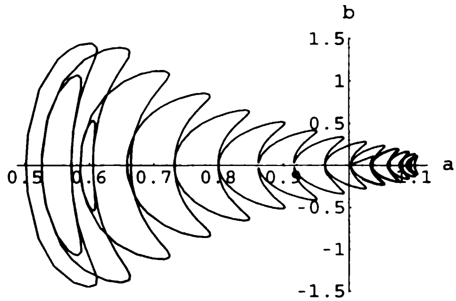

Figure 7.3: Phase portrait of variational equations with cubic-quintic nonlinearity for periodically varying diffraction with parameters $d_0 = 1$, $d_1 = 3.5$, $\Omega = 50, N = 14.717$.

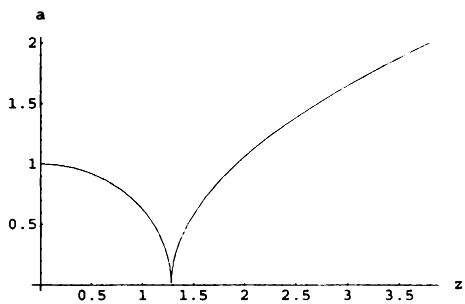

Figure 7.4: Variation of width $a(z)$ for constant diffraction with cubic-quintic nonlinearity

7.4 Light bullets

A confined wave packet in (3+1) dimension represents the extension of a self-trapped optical beam into the temporal domain and is known as Spatiotemporal soliton (STS). STS have been found to be useful as information carriers in communication and as energy sources, and have also been proposed for the design

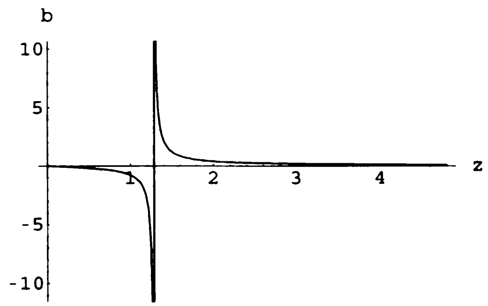

Figure 7.5: Variation of chirp $b(z)$ for constant diffraction with cubic-quintic nonlinearity

of optical switches and logic gates for all-optical devices [146] and hence the models that allow stable three-dimensional(3D) propagation of STS is a topic of growing interest due to its potential applications. However, to date a fully localized STS in three dimensions (3D) have not yet been found in an experiment. The stabilization of a 3D soliton in a Kerr medium by a rapidly oscillating dispersion coefficient using the numerical simulations and a variational method has been studied earlier [147]. Three dimensional light bullets collapses even in cubic-quintic medium when the coefficients of diffraction and dispersion are constants. In this work, we have formulated a model with periodically varying diffraction and dispersion for the stabilization of three dimensional light bullets in cubic-quintic medium.

The propagation of a continuous linearly polarized laser beam, in a nonlinear cubic quintic medium is given by Eq.(7.1).

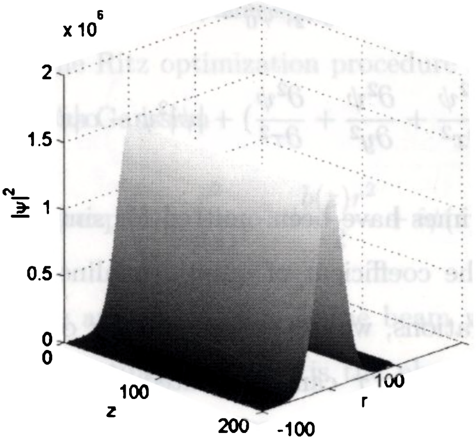

Figure 7.6: Evolution of a two dimensional spatial soliton according to numerical solution of equation (7.1) with periodically varying diffraction. The parameters of diffraction are $d_0 = 1$, $d_1 = 3.5$, $\Omega = 50, N = 14.717$.

If we include both the temporal and spatial effects with coefficients of varying dispersion and diffraction, Eq.(7.1) gets modified as

$$\begin{aligned} & 2ik\frac{\partial \psi}{\partial z} + (d(z)\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial \tau^2}) \\ & +2kk_0n_2|\psi|^2\psi - 2kk_0n_4|\psi|^4\psi = 0, \end{aligned} \tag{7.17}$$

where $d(z)$=d_0+$d_1(z)$. Normalizing (7.17) as $z' = \alpha z/z_0$, $x' = \sqrt{\alpha}x/r_0$, $y' = \sqrt{\alpha}y/r_0$, $\tau' = \sqrt{\alpha}\tau/r_0$, $\psi'^2 = \psi^2/\alpha\psi_0^2$ with $z_0 =$

$n_4/2kk_0n_2^2$, $r_0 = \sqrt{n_4/2kk_0n_2^2}$, $\psi_0^2 = n_4/n_2$, we obtain

$$i\frac{\partial\psi}{\partial z} + d(z)(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial \tau^2}) + |\psi|^2\psi - \alpha|\psi|^4\psi = 0. \quad (7.18)$$

In Eq.(7.18) primes have been omitted for simplicity. The parameter α is the coefficient of quintic nonlinearity. For analytical considerations, we use spherical polar coordinates. The co-moving coordinate τ can be treated on the same footing as spatial coordinate. If we introduce spatiotemporal radius $r = (x^2 + y^2 + t^2)^{\frac{1}{2}}$, Eq.(7.18) can be written as

$$i\frac{\partial\psi}{\partial z} + d(z)(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial\psi}{\partial r})) + |\psi|^2\psi - \alpha|\psi|^4\psi = 0. \quad (7.19)$$

where $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial \tau^2}) = (\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}))$, for spherically symmetric case. The coefficient $d(z)$ is periodic modulation of dispersion and diffraction and has the form $d(z)=d_0+d_1\sin(\Omega z)$ [100]. We follow a variational technique to describe the beam evolution based on Lagrangian formalism of classical mechanics. The Lagrangian density generating Eq.(7.19) is

$$\begin{aligned} L(\psi) &= \frac{ir^{D-1}}{2}\left(\frac{\partial\psi}{\partial z}\psi^* - \psi\frac{\partial\psi^*}{\partial z}\right) - d(z)r^{D-1}|\frac{\partial\psi}{\partial r}|^2 \\ &+\frac{1}{2}r^{D-1}|\psi|^4 - \frac{\alpha}{3}r^{D-1}|\psi|^6, \quad (7.20)\end{aligned}$$

where D is the dimension of the equation and $D = 3$ in this case. Following the Ritz optimization procedure let us assume the initial profile as Gaussian

$$\psi(r,z) = A(z)\exp\left(-\frac{r^2}{2a(z)^2} + i\frac{b(z)r^2}{2} + i\phi(z)\right). \qquad (7.21)$$

where $A(z)$ is the amplitude, $a(z)$ is the beam width, $b(z)$ is the wave front curvature, and $\phi(z)$ is the phase of the wave. Following the standard procedure, we insert the trial function into the expression for Lagrangian density and calculate the effective Lagrangian as

$$L_{eff} = C_D \int_0^\infty L(\psi)dr, \qquad (7.22)$$

where $C_D{=}4\pi$ in three dimensional case. The effective Lagrangian is obtained as

$$\begin{aligned} L_{eff} \;=\; & -\pi^{\frac{3}{2}}A^2a^3\phi_z - \frac{3}{4}\pi^{\frac{3}{2}}A^2a^5b_z - \frac{3}{2}d(z)A^2a\pi^{\frac{3}{2}} \\ & -\frac{3}{2}d(z)A^2a^5b^2\pi^{\frac{3}{2}} + \frac{1}{4\sqrt{2}}A^4a^3\pi^{\frac{3}{2}} \\ & -\frac{\alpha}{6\sqrt{3}}A^6a^3\pi^{\frac{3}{2}}. \end{aligned} \qquad (7.23)$$

Varying L_{eff} with respect to ϕ:

$$\frac{\delta L_{eff}}{\delta \phi} = 0 \Rightarrow \frac{\partial L_{eff}}{\partial \phi} - \frac{d}{dz}\left(\frac{\partial L_{eff}}{\partial\left(\frac{d\phi}{dz}\right)}\right) = 0, \tag{7.24}$$

yields,

$$\pi^{\frac{3}{2}} A^2 a^3 = N, \tag{7.25}$$

where N is the conserved quantity associated with the power of the beam.

$$\begin{aligned} \frac{L_{eff}}{N} &= -\phi_z - \frac{3}{4}a^2 b_z - \frac{3d(z)}{2a^2} - \frac{3}{2}d(z)a^2b^2 \\ &\quad + \frac{N}{4\sqrt{2}a^3\pi^{\frac{3}{2}}} - \frac{\alpha N^2}{6\sqrt{3}a^6\pi^3}. \end{aligned} \tag{7.26}$$

A set of evolution equations can be derived from Eq.(7.26) taking into account that the variation with respect to the unknowns in the initial profile is equal to zero. Varying Eq.(7.26) with respect to $a(z)$ and $b(z)$ yields the following equations,

$$\frac{da}{dz} = 2abd(z), \tag{7.27}$$

and

$$\frac{db}{dz} = \frac{2d(z)}{a^4} - 2d(z)b^2 - \frac{N}{2\sqrt{2}\pi^{\frac{3}{2}}a^5} + \frac{2\alpha N^2}{3\sqrt{3}\pi^3 a^8}. \tag{7.28}$$

Using Eqs.(7.27)and (7.28) we obtain a closed form evolution equation for width as

$$\frac{d^2a}{dz^2} = \frac{4d(z)^2}{a^3} - \frac{d(z)N}{\sqrt{2}\pi^{\frac{3}{2}}a^4} + \frac{\alpha d(z)N^2}{3\sqrt{3}\pi^3a^7} + \frac{da}{dz}\frac{d(d(z))}{dz}\frac{1}{d(z)}. \quad (7.29)$$

Rearranging (7.29) we obtain that

$$\frac{d}{dz}\left(\frac{1}{d(z)}\frac{da}{dz}\right) = -\frac{\partial U}{\partial a}, \quad (7.30)$$

where U is given by

$$U = \frac{2d(z)}{a^2} - \frac{N}{3\sqrt{2}\pi^{\frac{3}{2}}a^3} + \frac{\alpha N^2}{18\sqrt{3}\pi^3a^6}. \quad (7.31)$$

Now, Hamiltonian $H(a, \frac{da}{dz}, z)$ is given as

$$H(a, \frac{da}{dz}, z) = \frac{1}{d(z)}\frac{1}{2}(\frac{da}{dz})^2 + U(a, z). \quad (7.32)$$

When $d(z) = d_0$, $H(a, \frac{da}{dz}, z) \to \infty$, when $a \to 0$ i.e, when the beam width decreases intensity increases and intensity becomes infinitely large as the size of the beam diminishes leading to spatiotemporal collapse. When $d(z)$ is a periodic function, i.e., $d(z)=d_0+d_1\sin(\Omega z)$, Eq.(7.29) can be treated analytically by means of Kapitsa averaging method. We consider the dynamics of soliton beam by separating into two parts: a rapidly oscillating part with small amplitude and a slow, smooth varying part. The width of the soliton is then represented as $a(z) = a_0(z) + \rho(z)$, where $a_0(z)$ is the slowly varying part and $\rho(z)$ is the rapidly varying part with a zero mean value. Substituting $a(z)$ in Eq.(7.29) and separating the resulting equation into rapidly varying and slowly varying parts:

$$\frac{d^2\rho}{dz^2} = 4d_1 \sin(\Omega z)/a_0^3. \tag{7.33}$$

$$\frac{d^2a_0}{dz^2} = \frac{4d_0}{a_0^3} - \frac{12d_1\overline{\sin(\Omega z)\rho(z)}}{a_0^4} - \frac{N}{\sqrt{2\pi}\pi a_0^4} + \frac{\alpha N^2}{3\sqrt{3}\pi^3 a_0^7}, \tag{7.34}$$

where the overline indicates the average value. Integrating Eq.(7.33) yields

$$\rho(z) = -\frac{4d_1 \sin(\Omega z)}{\Omega^2 a_0^3}. \tag{7.35}$$

Substituting Eq.(7.35) in Eq.(7.34) we obtain the equation of motion for the slowly varying part as

$$\frac{d^2a_0}{dz^2} = \frac{4d_0}{a_0^3} + \frac{24d_1^2}{\Omega^2 a_0^7} - \frac{N}{\sqrt{2\pi}\pi a_0^4} + \frac{\alpha N^2}{3\sqrt{3}\pi^3 a_0^7}. \tag{7.36}$$

i.e,

$$\frac{d^2a_0}{dz^2} = -\frac{\partial}{\partial a_0}\left(\frac{2d_0}{a_0^2} + \frac{4d_1^2}{a_0^6\Omega^2}\right) - \frac{N}{3\sqrt{2\pi}\pi a_0^3} + \frac{\alpha N^2}{18\sqrt{3}\pi^3 a_0^7}. \tag{7.37}$$

Hence the effective potential U for the system is given by

$$U = \frac{2d_0}{a_0^2} + \frac{4d_1^2}{a_0^6\Omega^2} - \frac{N}{3\sqrt{2\pi}\pi a_0^3} + \frac{\alpha N^2}{18\sqrt{3}\pi^3 a_0^6} \qquad (7.38)$$

The periodically varying diffraction and dispersion introduces an additional repulsive potential near the small values of width $\sim 1/a^6$, which counteracts attractive potential and the exact balance between these terms give rise to a stable state. The plot of potential function given by Eq.(7.38) is as shown in Figure 7.7. The potential function has a well structure which implies that light bullets propagate with unchanged form as a consequence of exact balance between repulsive and attractive forces. But for constant diffraction and dispersion the potential function has no minima and stable light bullets cannot be formed. The stabilization mechanism of the spatiotemporal soliton in this case can be compared to that of a pendulum with an oscillating pivot. A pendulum with an oscillating pivot stabilizes when its bob is situated above the pivot. This surprising phenomenon is due to a net stabilizing force produced by alternating stabilizing and destabilizing forces at a frequency much faster than the natural oscillation frequency of the fixed pivot pendulum. In the present case, when there is an exact balance between repulsive term arising from periodically varying coefficients and the forces induced by competing nonlinearity, stable bright light bullets are formed.

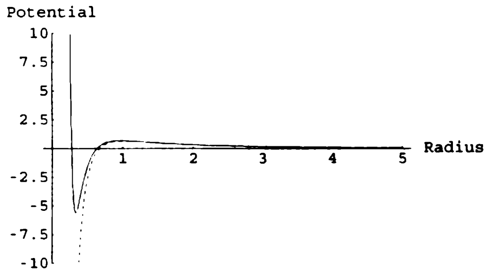

Figure 7.7: qualitative plot of potential function for $N = 31.7826$ $d_0 = 1$, $d_1 = 3.5$, $N = 31.7826$, $\alpha = .001$.(solid line shows the plot of potential function given by (7.38) and dotted line shows the plot of potential function for constant diffraction and dispersion)

7.5 Numerical analysis

The coupled differential Eqs.(7.27), (7.28) and the partial differential Eq.(7.17) has been studied numerically to analyze the dynamics of light bullets in cubic-quintic media with periodically varying coefficients. The differential equations (7.27) and (7.28) have been solved for constant and periodically varying dispersion and diffraction. The variation of intensity with propagation distance for constant diffraction and dispersion is as shown in Figure 7.8. Intensity of the spatiotemporal soliton increases with propagation distance and the beam collapses after a finite propagation distance.

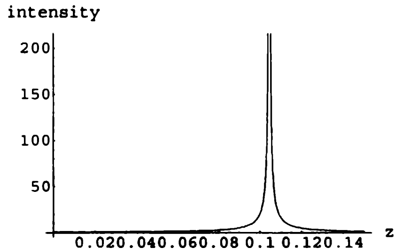

Figure 7.8: Variation of intensity of light bullet with propagation distance in cubic-quintic medium as obtained from simulation of variational equations (7.27 and 7.28) for constant dispersion and diffraction i.e. $d_0 = 1$ and $d_1 = 0$, $N = 31.7826$ $\alpha = .001$.

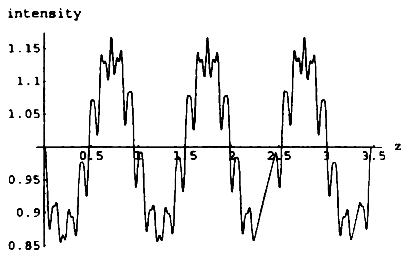

Figure 7.9: Variation of intensity of light bullet with propagation distance in cubic-quintic medium as obtained from simulation of variational equations (7.27 and 7.28) for periodic dispersion and diffraction i.e. $d_0 = 1$ and $d_1 = 3.5, \Omega = 50$ $N = 31.7826$ $\alpha = .001$.

When $d(z){=}d_0{+}d_1 \sin(\Omega z)$, intensity with propagation distance is as shown in Figure 7.9. Intensity of the spatiotemporal soliton does not increase above 1.15 and does not decrease below 0.85 due to the effect of periodically varying coefficients. i.e., periodically varying diffraction and dispersion together with self focusing cubic and self defocusing quintic nonlinearity stabilize three dimensional light bullets against collapse for higher incident energies.

We have studied the partial differential equation numerically using finite difference beam propagation method(FD-BPM). The initial state was taken as per the ansatz in Eq.(7.21), for the numerical simulation of partial differential equation. The numerical parameters of simulation has been chosen so as to fit the experimental configurations. The nonlinear parameters chosen are $n_0 = 3.3$, $n_2 = 1.5\times10^{-13}cm^2/W$, $n_4 = 5\times10^{-23}cm^4/W^2$ at $1.55\mu m$ wavelength and $n_0 = 1.6755$, $n_2 = 2.2\times10^{-12}cm^2/W$, $n_4 = 8 \times 10^{-22}cm^4/W^2$ at $1.6\mu m$ [150]. The parameters of dispersion and diffraction are $d_0 = 1$, $d_1 = 3.5$, $\Omega = 50$. The normalized beam profile at the output face after five diffraction lengths of travel through the medium is as shown in Figure 7.10 The axial section profile of the beam evolution in the cubic-quintic medium is as shown in Figure 7.11. If the periodic modulation of dispersion and diffraction were not applied, the initial wave form would have collapsed. Numerical simulations of variational equations and partial differential equation have shown that average intensity of light bullets remain constant as they propagate through the cubic-quintic medium with periodically varying coefficients of dispersion and diffraction.

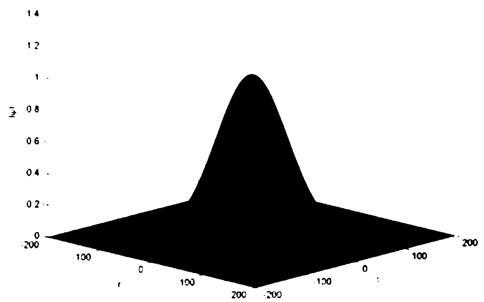

Figure 7.10: Normalized intensity distribution after five diffraction lengths of travel through the cubic-quintic medium

7.6 Conclusion

Stabilization of (2+1) dimensional spatial solitons and (3+1) dimensional spatiotemporal solitons in cubic-quintic media has been analyzed in this chapter. A diffraction managed model in cubic-quintic media has been formulated and analyzed. Variational approach has been used to derive a set of ordinary differential equations which describe the optical beam evolution. We have simulated variational equations and partial differential equation. A dispersion managed model with diffraction management has been proposed for the stabilization of three dimensional spatiotemporal soilton in bulk cubic-quintic media The cubic-quintic nonlinear Schrodinger equation with periodically varying dispersion and diffraction has been studied analytically

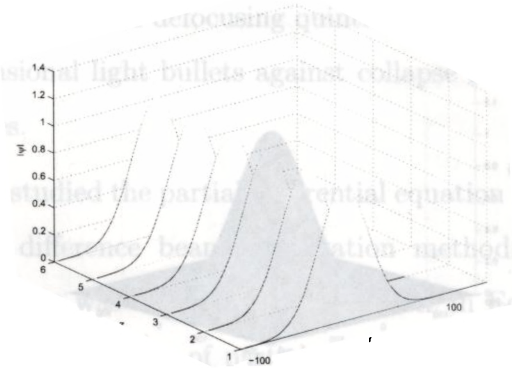

Figure 7.11: numerically simulated stable 3D light bullet generation in cubic-quintic media with periodically varying dispersion and diffraction

using variational analysis and Kapitsa averaging method. The averaged equation for soliton width successfully describes the long time evolution of the soliton and the stability of the spatiotemporal soliton is inferred from the plot of the potential function. Analytical and numerical studies have shown that diffraction management together with self focusing cubic and self defocusing quintic nonlinearity stabilize the two dimensional spatial soliton against collapse while periodically varying coefficients of diffraction and dispersion stabilizes the spatiotemporal soliton in cubic-quintic media for higher incident energies.

Chapter 8

Results and Conclusion

8.1 Results and Conclusion

The thesis presents a study on optical soliton propagation in inhomogeneous single mode and coupled fibres and the stabilization of multidimensional spatial solitons.

An inhomogeneous optic fibre system with varying dispersion, frequency chirping within the integrable limit has been analyzed. The exact two soliton solution for the nonlinear Schrodinger equation has been constructed using a recursive method and Backlund transformation technique. As the pulse propagates through the fibre, the pulse gets amplified and compressed which supports multi soliton pulse compression technique. In practical applications, the integrable system is not an adequate model due to the restrictions imposed on the system to make it integrable.

The inhomogeneous optic fibre system with varying dispersion is not in general integrable. The nonintegrable inhomogeneous optic fibre has been studied. The inhomogeneous optic fibre system has been studied for different dispersion decreasing GVD parameters using variational analysis and numerical methods. The different GVD parameters chosen are exponential, Gaussian and hyperbolic. As the pulse propagates through the fibre, the pulse gets compressed and amplified, which is similar to the adiabatic compression of solitons. Numerical and variational studies reveal that pulse compression is more in the case of Gaussian and hyperbolic GVD profiles than exponential profile. The pulse compression and amplification is mainly due to the interplay between the inherent gain of the dispersion decreasing profile and the effective phase modulation. The pulse compression for Gaussian and hyperbolic profile is very promising and can be used for the generation of ultra short pulses while dispersion decreasing fibers with exponential dispersion profile can be used for transmission of ultrashort pulses over relatively long lengths. In a constant dispersion fibre, solitons broaden as they loose energy because of weakening of nonlinear effects. The width of a fundamental soliton can be maintained inspite of fibre losses, if GVD decreases exponentially.

The study has been extended to coupled systems. Soliton switching in an asymmetric coupler with varying dispersion has been analyzed. The coupled nonlinear Schrodinger equations which describe the pulse coupling in nonlinear directional coupler has been studied analytically and numerically. Variational analysis has been used to derive a set of coupled differential equations which describe switching dynamics. The coupled differential equations have been studied numerically for periodically varying dispersion. Finite difference beam propagation method has been used for direct partial differential equation simulation. The study has shown that for low input powers, pulse switches from one core to other and when input power increase pulse remains in the first core. The energy transmission characteristics of a half beat length coupler have been calculated and plotted as a function of input peak power. The critical switching power can be reduced for an asymmetric coupler with varying dispersion.

We have explored the existence of two dimensional spatial soliton in Kerr-media with periodically varying diffraction and nonlinearity. Variational approach has been used to derive a set of ordinary differential equations which describe the optical beam evolution. Analytical and numerical studies have shown that the periodic force arising due to the periodically varying diffraction and nonlinearity stabilize the two dimensional spatial solitons. When the coefficient of diffraction and nonlinearity are constants, the beam collapses after propagating through a finite distance. When diffraction is managed keeping nonlinearity constant, stable solitons are formed for lower energy of the incident beam, but for larger energies, the beam decays. When nonlinearity is managed, keeping diffraction constant, the beam collapses for larger energies of the incident beam. Analytical and numerical studies have shown that the diffraction management and nonlinearity management can stabilize the beam against decay or collapse providing undisturbed propagation even for larger energies of the incident beam.

The stabilization of spatiotemporal solitons in Kerr media with periodically varying dispersion, diffraction and nonlinearity has been analyzed using variational approach and Kapitsa averaging method. Light bullets get stabilized in Kerr media due to the combined effect of periodically varying coefficients of dispersion, diffraction and nonlinearity.

Stabilization of (2+1) dimensional spatial solitons and (3+1) dimensional spatiotemporal solitons in cubic-quintic media has also been analyzed. A diffraction managed model in cubic-quintic media has been formulated and analyzed for the stabilization of (2+1) dimensional spatial solitons . Variational approach has been used to derive a set of ordinary differential equations which describe the optical beam evolution. We have simulated variational equations and partial differential equation. A dispersion managed model with diffraction management has been proposed for the stabilization of three dimensional spatiotemporal soilton in bulk cubic-quintic media. The cubic-quintic nonlinear Schrodinger equation with periodically varying dispersion and diffraction has been studied analytically using variational analysis and Kapitsa averaging method. The averaged equation for soliton width successfully describes the long time evolution of the soliton and the stability of the spatiotemporal soliton is inferred from the plot of the potential function. Analytical and numerical studies have shown that diffraction management together with self focusing cubic and self defocusing quintic nonlinearity stabilize the two dimensional spatial soliton against collapse while periodically varying coefficients of diffraction and dispersion stabilizes the spatiotemporal soliton in cubic-quintic media for higher incident energies.

8.2 Future Prospects

All optical switching devices have been an active field of research due to their potential for ultra fast signal processing. Optical solitons have been considered as the most suitable candidates for application in all-optical switching due to their unique property of propagation without distortion and spreading. Also, the nonlinear coupling has been considered for the purposes of all-optical switching and all-optical ultra fast logic functions. The coupled equations have to be studied through numerical modeling inorder to analyze logic functions. A periodic array of wave guide creates a novel kind of device in which new kinds of spatial solitons can be generated. The properties of spatial solitons in wave guide arrays have to be analyzed in the frame work of a set of coupled equations. Such set of coupled equations are known as discrete nonlinear Schrodinger equations. The localized solutions of discrete nonlinear Schrodinger equation, known as discrete solitons are used for all optical switching. The study of coupled systems in wave guide arrays and other physically relevant models are also to be explored. Soliton pulse compression in graded index Kerr media is another area where the present work can be extended.

References

[1] N.J. Zabusky and M.D. Kruskal. *Rev.Lett.* **15**, 240 (1965).

[2] M. Wadati, *Pramana J. Phys.* **57**, 841 (2001).

[3] D.J. Korteweg and C. de Vries, *Philos.Mag.* **39**, 422 (1898).

[4] C. S. Gardner et al., *Phys. Rev.Lett.* **19**, 1095 (1967).

[5] V. B. Zakkarov and A. B. Shabat, *Sov. phy. JETP* **34**, 62 (1972).

[6] Y.S. Kivshar and G. P. Agrawal. *Optical solitons: From Fibers to Photonic Crystals* (San Diego: Academic Press 2003).

[7] A. Hasegawa and F.D Tappert, *Appl. Phys Lett.***23**, 142 (1973).

[8] L. F. Mollenauer et al., *Opt.Lett.* **8**, 289 (1983).

[9] G.P. Agrawal, *Nonlinear Fiber Optics*, (San Diego: Academic press,1995).

[10] J. P. Gordon and H. A. Haus, *Opt. Lett.* **11**, 665 (1986).

[11] K. J. Blow and N. J. Doran, *Elect. Lett.* **19**, 429 (1983).

[12] J. P. Gordon. *Opt. Lett.* **8**, 596 (1983).

[13] A. Hasegawa and Y. Kodama, *Solitons in Optical Communications* (Oxford: Oxford University Press, 1995).

[14] I. Iannone et al., *Nonlinear Optical Communication Networks* (New York: Wiley, 1998).

[15] P.V. Mamyshev, *Optical solitons theory and experiment* J.R.Taylor Ed.(Cambridge: Cambridge University Press, 1992).

[16] R.Y. Chaio, E. Garmier and C.H. Townes, *Phys.Rev.Lett.* **13**, 479 (1964).

[17] J.S Aitchison et al., *Opt.Lett.* **15**, 471 (1990).

[18] S.A. Akhamanov, A.P. Sukhorukov and R.V. Khokhlov, *Usp.Fiz.Nauk.* **93**, 19 (1967).

[19] O. Svelto, *in Progress in Optics*, ed. E. Wolf(North-Holand: Amsterdam, 1974).

[20] V.E. Zakharov et al., *Theory of solitons: The Inverse Scattering Transform* (Nauka, Moscow, 1980).

[21] Berge L, *Phys. Rep.* **303**, 260 (1998).

[22] M. Segev and N. ChristodoulidesD, *Opt. Photon. News* **13**, 70 (2002).

[23] M. Segev et al., *Phys. Rev.Lett.* **68**, 923 (1992).

[24] G. Duree et al., *Phys. Rev. Lett.* **74**, (1995).

[25] G. Duree et al., *Phys. Rev. Lett.* **71**, 533 (1993).

[26] M.Mitchel and M.Segev, *Nature* **387**, 880 (1997).

[27] J.W. Fleischer et al., *Phys. Rev. Lett.* **92**, 123904 (2004).

[28] D.N. Neshev et al., *Phys. Rev. Lett.* **92**, 123903 (2004).

[29] A.E. Kaplan et al., *Phys.Rev.Lett.* **55**, 1291 (1985).

[30] F.Kh. Abdullaev, S. Darmanyan and P. Khabibullaev, *Optical solitons* (Berlin: Spinger, 1991).

[31] G.P. Agrawal, *Applications of Non Linear Fibre Optics* (San Diego: Academic, 2001).

[32] J.T. Ong et al., *IEEE J. Quan. Elect.* **29**, 1701 (1993).

[33] K.A. Ahmed, K.C. Chan and H.F. Liu, *IEEE J. Quantum Electron* **1**, 592 (1995).

[34] C.F. Mollenauer, R.H. Stolen and J.P. Gordon, *Phys Rev. Lett.* **45**, 1095 (1980).

[35] S.V. Cherinkov et al., *Elect. Lett.* **28**, 1842 (1992).

[36] S.V. Cherinkov et al., *Opt.Lett.* **18**, 476 (1993).

[37] Y. Kodama and A. Hasegawa, *Opt.Lett.* **7**, 285 (1982).

[38] M.N. Vinoj, V.C. Kuriakose and K. Porsezian, *Chaos Solitons Fract.* **12** 2569 (2001).

[39] K.J. Nakkeeran,*J Phys. A: Math. Gen.* **34**. 5111 (2001).

[40] M.J. Ablowitz et al., *stud. Appl. Math.* **53**. 249 (1974).

[41] V.B. Zakharov and L.D. Fadeev, *Funt.Anal.Appl.* **5**, 280 (1972).

[42] A.C. Newell and J. P. Moloney, *Nonlinear Optics* (Redwood City:Addison-Wesley 1992).

[43] K. Konno and M. Wadati, *Pro. Theor. phy* **53**, 1652 (1975).

[44] H.H. Chen, *Phys Rev. lett.* **33**, 925 (1974).

[45] V.I. Karpman, *Phys. scr.* **20**, 462 (1978).

[46] V. Bargmann, *Rev. Mod Phy.* **21**, 488 (1948).

[47] A.A. Rangwala and A.R. Jyoti, *Phy lett. A* **112**, 188 (1985).

[48] K.Tai and A.Tomita, *Appl.Phys.Lett.* **48**, 1033 (1986).

[49] E.M. Daniov et al., *IEEE J.Quan. Elect.* **25**, 828 (1989).

[50] M. Suzuki et al., *Elect. Lett.* **31**, 2027 (1995).

[51] K.A.Ahmed et al., *Elect. Lett.* **29**, 57 (1993).

[52] E.M. Dianov et al., *Opt.Lett.* **14**, 1008 (1989).

[53] M.N. Vinoj and V.C. Kuriakose, *J. Opt. A:Pure and Appl. Opt.* **6**, 63 (2004).

[54] J.D. Moores, *Opt. Lett.* **21**, 555 (1996).

[55] S.V. Cherinkov et al., *Elect.Lett.* **28**, 1842 (1992).

[56] S.V. Cherinkov et al., *Elect.Lett.* **28**, 1210 (1992).

[57] S.V. Cherinkov et al., *Elect.Lett.* **28**, 931 (1992).

[58] K. Tajima, *Opt. Lett.* **12**, 54 (1987).

[59] R.J. Essaimbre and G.P. Agrawal, *Opt. Lett.* **21**, 1770 (1996).

[60] F.M. Knox, W. Forysiak, and N.J. Doran, *J. Lightwave Technol.* **13**, 1955 (1995).

[61] A. Maruta, Y. Nonaka and T. Inoue, *Elect.Lett.* **37**, 1357 (2001).

[62] A. Maruta et al., *IEEE J.Select. Topics on Quan. Elect.* **8**, 640 (2002).

[63] H. Goldstein, Classical Mechanics, (Redwood City, CA: Addison-Wesley, 1950).

(Cambridge university press, NewYork, 1992).

[64] D. Anderson, *Phys.Rev.A* **27**, 3135 (1983).

[65] F.Kh. Abdullaev et al., *Phys Rev.A* **67**, 013605 (2003).

[66] F.Kh. Abdullaev and J.G. Caputo, *Phys Rev.E* **58**, 6637 (1998).

[67] T. Iami et al., *Elect. Lett.* **28**, 1484 (1992).

[68] M.I. Hayee and A.E. Willner, *IEEE Photon.Technol. Lett.* **11**, 991 (1999).

[69] M. Murakami et al., *J.Lightwave Technol.* **18**, 1197 (2000).

[70] V.I. Karpman and E.M.Krushkal, *Sov. Phys. JETP* **8**, 277 (1969).

[71] W.P.Zeng, *J.Comp.Math.* **17**, 133 (1999).

[72] Q.Chang, E.Jia and W.Son, *J.Comp.Phys.* **148**, 397(1999).

[73] K.J. Blow, *Optical solitons Theor.and exp.challenges*, ed. K.Porsezian and V.C.Kuriakose (Berlin: springer, 2002).

[74] A.J.Stentz, R.W.Boyd and A.F.Evans, *Opt. Lett.* **20**, 1770 (1995).

[75] A.A. Maier, *Sov.J.Ouan. Elect.* **12**, 1490 (1982).

[76] S.M. Jenson, *IEE J.Quan. Elect.* **18**, 1580 (1982).

[77] H.G. Winful, *Appl.Phys.Lett.* **47**, 213 (1983).

[78] C. Pare', M. Florjanezyk, *Phys.Rev.A* **41**, 6287 (1990).

[79] R.H. Enns, R. Fung and S.S. Rangekar, *IEE J.Quan. Elect.* **27**, 252 (1991).

[80] N. Akhmediev and A.A. Ankiewicz, *Phys. Rev. Lett.* **70**, 2395 (1993).

[81] P.L. Chu, B.A. Malomed and G.D. Peng *J.Opt.Soc.Am.B* **10**, 1379 (1993).

[82] I.M. Uzunov et al., *Phys.Rev.E* **51**, 2527 (1995).

[83] T.I. Lakoba, D.J. Kaup and B.A. Malomed, *Phys. Rev.E* **55**, 6107 (1997).

[84] B.A. Malomed, *Phys. Rev.E* **51**, R864 (1995).

[85] P.B. Hansen et al., *Opt.Commun.* **119**, 178 (1995).

[86] B.A. Malomed et al., *Phys.Rev.E* **53**, 4084 (1996).

[87] D.J. Kaup, T.I. Lakoba, B.A. Malomed, *J.Opt.Soc.Am.B* **14**, 1199 (1997).

[88] D.J. Kaup and B.A. Malomed, *J.Opt.Soc.Am.B* **15**, 2838 (1998).

[89] K. Tajima, *Opt. Lett.* **12**, 54 (1987).

[90] B.A. Malomed, *Physica Scripta* **19**, 341 (1993).

[91] N.J. Doran, W. Forysiak and F.M. Knox, *Opt. Lett.* **19**,174 (1994).

[92] Y.S. Kivshar, *Optics Lett.* **18**, 7 (1993).

[93] N.N. Akheimediev and A.V. Buryak, *J.Opt.Soc.Am.B* **11**, 804 (1994).

[94] A. Gourdin and M. Boumahrat *Applied Numerical Methods* (New Delhi: Prentice Hall, 1996)

[95] R.Y. Chaio, *Spatial solitons*, eds. S Trillo and W Torruellas, (Berlin: Springer verlag, 2001).

[96] B.A. Malomed et al., *J.Opt.B: Quant. Semiclass.Opt.* **7**, R53 (2005).

[97] N.J. Smith et al., *Elect. Lett.* **32**, 54 (1996).

[98] A. Hasegawa and Y. Kodama, *Solitons in Optical Communications* (Oxford:Oxford University Press, 1995).

[99] M.J. Ablowitz and Z.H. Musslimani, *Phys.Rev.Lett.* **87**, 254102 (2001).

[100] F.Kh. Abdullaev, B. Baizakov and M. Salerno, *Phys.Rev.E* **68**, 066605 (2003).

[101] F.Kh. Abdullaev, J.G. Caputo and N.J. Smith, *Phys.Rev.E* **50**, 1552 (1994).

[102] I. Towers and B.A. Malomed, *J.Opt.Soc.Am.B* **19**, 537 (2002).

[103] H. Saito and M. Ueda, *Phys.Rev.Lett.* **90**, 040403 (2003).

[104] D. Mihalache et al., *Phys.Rev.E* **66**, 016613 (2002).

[105] L.C. Crasovan, B.A. Malomed and D. Mihalache, *Phys.Rev.E* **63**, 016605 (2000).

[106] D. Mihalache et al., *Phys.Rev.Lett.* **88**, 073902 (2002).

[107] G. Mohanachandran, V.C. Kuriakose and K. Babu Joseph, *Pramana, j. of phys.* **46**, 305 (1996).

[108] B.A. Malomed, *Prog.Opt.* **43**, 71 (2002).

[109] S.K. Turitsyn et al., *Phys. Rev. E* **58**, R48 (1998).

[110] B.A. Malomed, D.F. Parker and N. Smyth, *Phys. Rev. E* **48**, 1418 (1993).

[111] N.N. Akhmediev and A. Ankiewicz, *Solitons Nonlinear pulses and beams* (London: Chapman and Hall 1997).

[112] L.D. Landau and E.M. Lifshitz, *Mechanics* (Oxford: Pergamon Press, 1976).

[113] F.Kh. Abdullaev and R.A. Kraenkel, *Phys. Lett. A* **272** 395 (2000).

[114] W.H. Press et al., *The Art of Scintific Computing*(Cambridge:Cambridge University Press 1996).

[115] B.A. Malomed et al., *J. Opt. B: Quantum Semiclass. Opt* **7**, R53(2005).

[116] Y. Silberberg, *Opt.Lett.* **15**, 1282 (1990).

[117] L. Berge, *Phys. Rep.* **303**, 260 (1998).

[118] N. Akhmediev and J.M. Soto-Crespo, *Phys. Rev. A* **47**, 1358 (1993).

[119] D.E. Edmundson, *Phys. Rev. E* **55**, 7636 (1997).

[120] J.T. Manassah, P.L. Baldeck and R.R. Alfano, *Opt. Lett.* **13**, 1090 (1988).

[121] K. Hayata and M. Koshiba, *Phys. Rev. Lett.* **71**, 3275 (1993).

[122] B.A. Malomed et al., *Phys. Rev. E* **56**, 4725 (1997).

[123] D. Mihalache et al., *Opt. Commun.* **159**, 129 (1999).

[124] S. Raghavan and G.P. Agrawal, *Opt. Commun.* **180**, 377 (2000).

[125] Y. Chen and J. Atai. *Opt. Lett.* **20** , 133 (1995).

[126] V. Skarka, V.I. Berezhiani and R. Miklaszewski, *Phys. Rev. E* **56**, 1080 (1997) .

[127] D. Mihalache et al.,*Opt.Commun.* **169**, 341 (1999).

[128] H. S. Eisenberg et al., *Phy. Rev. Lett.* **85**, 1864 (2000).

[129] M. Matuszewski et al., *Phys.Rev.E* **70**, 016603 (2004).

[130] S.K. Adhikari, *Phys. Rev. A* **69**, 063613 (2004).

[131] S.K. Adhikari, *Phys. Rev. E* **70**, 036608 (2004).

[132] K. Dimitrevski and E. Sevensson et al., *Phys. Lett. A* **68**, 369 (1995).

[133] W. Chao, *Opt.Commnu.* **175**, 239 (2000).

[134] D. Pushkarov and S. Tanev, *Opt.Commun.* **124**, 354 (1995).

[135] B. Lawrence et al., *Phys.Rev.Lett* **73**, 597 (1994).

[136] A. Desyatnikov, A. Maimistov and B.A. Malomed, *Phys.Rev.E* **61**, 3107 (2000).

[137] I. V. Barashenkov and A.Y. Panova, *Physica D* **69**, 114 (1993).

[138] C. Josserand and S. Rica, *Phys.Rev.Lett.* **78**, 1215 (1997).

[139] J. Atai and B.A. Malomed, *Phys. Lett. A* **284**, 247 (2001).

[140] M. Quiroga-Teixeiro and H. Michinel, *J.Opt.Soc.Am.B* **14**, 1697 (2004).

[141] I. Towers et al., *Phys.Lett.A* **288**, 292 (2001).

[142] C.P. Jisha, V.C. Kuriakose and K. Porsezian, *Phys.Rev.E* **71**, 056615 (2005).

[143] C.P. Jisha, V.C. Kuriakose and K. Porsezian, *Phys. Lett. A* **352**, 496 (2005).

[144] P. L. Kelly, *Phys. Rev. Lett.* **15** , 1005 (1965).

[145] E. Kutnetsov, V.E. Zakharov and E. Vladimir Eds. *Wave Collapse* (Singapore: World Scientific, 1990).

[146] W.J. Firth et al., *JOSA B* **19**, 747 (2002).

[147] S.K. Adhikari et al., *Phys.Rev.E.* **71**, 016611 (2005) .

[148] C. Lin et al., *Opt. Lett* **5**, 476 (1980).

[149] I.R. Gabitov and S.K. Turitsyn, *Opt. Lett* **21**, 327 (1996).

[150] S. Saltiel, S.Tanev and A.D.Boardman, *Opt.Lett.* **22** 148 (1997).