STUDIES ON SOME ASPECTS OF WAVE PROPAGATION THROUGH NONLINEAR MEDIA

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DOCTOR OF PHILOSOPHY

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The White Rabbit put on his spectacles.

"Where shall I begin, please your Majesty?" he asked.

"Begin at the beginning" the King said gravely,

"and go on till you come to the end; then stop"

Lewis Carroll, 'Alice in Wonderland'

DECLARATION

I here by declare that the work presented in this thesis is based on the original work done by me under the guidance of Dr.V.C.Kuriakose in the Department of Physics, Cochin University of Science and Technology and has not been included in any other thesis submitted previously for the award of any degree.

Kochi-22,

January , 2000. S.G.Bindu 15 1 2000

CERTIFICATE

Certified that the work presented in this thesis is the bonafide work done by Ms. S.G.Bindu, under my guidance in the Department of Physics, Cochin University of Science and Technology and that this work has not been included in any other thesis submitted previously for the award of any degree.

15-1-2000

Kochi-22, Dr.V.C.Kuriakose

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PREFACE

The work presented in this thesis has been carried out by the author as a full time research scholar in the Department of Physics, Cochin University of Science and Technology during the period 1995-2000.

Nonlinearity is a charming element of nature and Nonlinear Science has now become one of the most important tools for the fundamental understanding of the nature. Solitons- solutions of a class of nonlinear partial differential equations - which propagate without spreading and having particle- like properties represent one of the most striking aspects of nonlinear phenomena. The study of wave propagation through nonlinear media has wide applications in different branches of physics.Different mathematical techniques have been introduced to study nonlinear systems.

The thesis deals with the study of some of the aspects of electromagnetic wave propagation through nonlinear media, viz. plasma and ferromagnets, using reductive perturbation method. The thesis contains 6 chapters.

In the first chapter a general introduction to nonlinear science, the standard methods for studying different nonlinear evolution equations such as Painleve analysis, Inverse Scattering Transform are explained. The methods adopted for carrying out the present work-the reductive perturbation method (RPM), the K- perturbation expansion method and the multiscale expansion method- are also presented in this chapter.

Chapter 2 deals with the study of the nonlinear wave propagation through a cold collisionless plasma in (1+1) dimensions and in (2+1) dimensions using the reductive perturbation method. In (1+1) dimension the appropriate NEE describing the wave propagation is found to be the modified Korteweg-deVries equation (mKdV) while in (2+1) dimensions the wave propagation is described by modified Kadomtsev-Petviashvili equation (mKP).The solutions of these equations have been obtained.

The stability analysis of the solitary traveling wave solution of the mKP equation is studied in chapter 3.

The fourth chapter of the thesis contains a study of the interaction between two solitary waves in plasma in (1+1) dimensions having different velocities and amplitudes.

The fifth chapter deals with the propagation of electromagnetic waves through a ferromagnet in the presence of a damping term in (2+1) dimensions.

The summary and conclusions of the present work are presented in chapter 6.

A part of the present work has been published and presented in Scientific seminars/Conferences:

1.*Nonlinear wave propagation through cold collision free plasma*, S.G.Bindu and V.C.Kuriakose, J.Non.Math.Phys**5**, 149-158, (1998)

2.*Solitons and electromagnetic waves through a cold collisionless plasma*, S.G.Bindu and V.C.Kuriakose, J.Phy.Soc.Jpn, **67**, 4031-4036, (1999).

3.*Modified Kadomtsev- Petviashvili equation in cold collisionless plasma*, S.G.Bindu and V.C.Kuriakose, Pramana-J.Phys.**52**, 39-47 (1999).

4.*Nonlinear wave propagation through a ferromagnet with damping in (2+1) dimensions, S.G.Bindu and V.C.Kuriakose* (To appear in Pramana-J.Phys.)

5.*Solitary wave interaction in a cold plasma*, S.G.Bindu and V.C.Kuriakose (communicated to J. Phy.Soc.Japan.)

6.*Stability analysis of modified Kadomtsev Petviashvili equation*, S.G.Bindu and V.C.Kuriakose (communicated to J. Phy.soc.Japan.).

7.*Nonlinear wave propagation through plasma in (1+1) dimensions*, S.G.Bindu and V.C.Kuriakose, presented in the **41st congress of ISTAM** ,CUSAT, Cochin, Dec.17-20, (1996).

8.*Nonlinear wave propagation through plasma in (2+1) dimensions*, S.G.Bindu and V.C.Kuriakose, presented in the **12th National symposium on plasma dynamics, PSSI, IPR**, Bhat, Ahmedabad, Dec.2-5, (1997).

9.*The stability analysis of modified Kadomtsev-Petviashvili equation*, S.G.Bindu and V.C.Kuriakose,(presented in the **International Conference on Nonlinear Dynamics**, Bharathidasan University,Tiruchirappilly, Feb.12-16,(1998)).

SYNOPSIS

The study of the theory of solitons and related issues of the construction of solutions to a wide class of nonlinear equations have attracted great attention of physicists and mathematicians during the past thirty years The concept of solitons has now become ubiquitous in modern nonlinear science and indeed can be found in various branches of physics,e.g. hydrodynamics, plasma physics, nonlinear optics, solid state physics, etc. Exciting and important discoveries have been made in the nonlinear dynamics of dissipative and conservative systems. There are different methods to study nonlinear systems. The soliton phenomenon and integrable nonlinear equations represent important and well established fields of theoretical physics,mathematical physics and applied mathematics.

Zabusky and Kruskal in their pioneering work on the Korteweg-deVries equation observed that the solutions of this equation exhibit particle like character and hence thes solutions were called solitons. Solitons are special type of solutions of the complete integrable nonlinear evolution equations (NEEs). NEEs having soliton solutions sh many special properties like infinite sequence of conservation laws, the Lie-Backl symmetries, multi soliton solutions Backlund transformations, reduction to ordi differential equations of Painleve type,etc. Further-more these equations may b tained via the compatibility of two associated linear operators. Therefore they c put in Lax form and equations are said to be exactly solvable.

Despite its importance and interest, the study of exactly solvable models is giving up its protagonist role to the investigation of perturbation of such models, to take into account of the deviations of real physical systems from ideal ones. Perturbations usually give rise to a drastic modification of the equations and many of them turn to be unsolvable. Hence several approximate analytical techniques have been proposed. In addition, the availability of fast computers has helped numerical computations. However, both approaches have achieved their best results when applied in co-operation. Thus we have a variety of physical, chemical, biological or technological problems that can be modeled by perturbed nonlinear equations.

Most of the wave motions are represented mathematically by nonlinear partial differential equations. The role of partial differential equations in the study of nonlinear wave propagation is important and a knowledge of the properties of their solutions, both qualitative and quantitative is of considerable importance when applications to physical problems are involved. Nonlinearity in waves manifests itself in a variety of ways. Nonlinear waves are more complicated than linear waves. Among the nonlinear partial differential equations of physical interests, an important role is played by the class of evolution equations which possess a series of remarkable features such as solitary wave solutions which arise from the balancing of nonlinear term with the dispersive term, which preserve the shape of the wave through nonlinear interaction.

The NEEs frequently arise as a result of the determination of the asymptotic behaviour of either a higher order equation or a complicated system of equations and they are then often called far field equations. To study the asymptotic behaviour of nonlinear dispersive waves, Gardner and Morikava introduced a scale transformation derived from the linearized asymptotic behaviour of long waves and they combined this transformation with a perturbation expansion of the dependent variables so as to describe nonlinear asymptotic behaviour. This type of perturbation has been developed and formulated in a general way and is now called the reductive perturbation method (RPM) by Taniuti and his collaborators. This method is a systematic way for the reduction of a fairly general nonlinear system to a single tractable nonlinear evolution equation describing the far field behaviour. The terms solitary waves and solitons are well known in condensed matter physics, nonlinear optics, plasma physics and particle physics. The study of propagation of nonlinear waves through disordered media can reveal many properties of the system.

One of the most important aspects of present-day plasma research is the emergence of nonlinear solitonic excitations. The knowledge of the properties of nonlinear electromagnetic waves in plasma is rather essential for understanding the salient features of Alfven like disturbances, which have been frequently observed in the solar wind as well as in the Earth's magnetosphere and ionosphere. The existence of magnetic field perturbations observed in the solar wind can not be explained by the exact magnetohydrodynamic theory. A transformation of the nonlinear self-consistent equations for a collisionless cold plasma with stationery ions, which makes it possible to obtain new classes of analytic solutions. The propagation of electromagnetic waves through a cold collisionless plasma is investigated considering plasma as an electron fluid. By using the RPM we have reduced the system of equations to a modified Korteweg-deVries equation (mKdV) in (1+1) dimensional case and to a modified Kadomtsev- Petviashvili (mKP) equation in (2+1) dimensions. The soliton solution of the mKdV equation can be found by using the inverse scattering transform (IST) method. The mKP equation is found to be nonintegrable in the sense of Painleve analysis. But we have found the solitary wave solution of the mKP equation.

A second category of problems occur in wave propagation which are significantly different from the situation described above is associated with the stability of the solitary wave solutions. The stability analysis of the solitary wave solutions of mKP equation is studied using the K- expansion method of Rowlands and Infeld. The method is carried out by assuming that the original nonlinear structure has undergone a long wave length perturbation. The growth rate of instability is also found out.

The interaction of solitary waves in cold collisionless plasma is studied using the multiscale expansion method and obtained a linear hyperbolic system of equations. The usual behaviour of soliton interaction (conservation of the shape, size, velocity and the appearance of a phase shift) is *apriori* not to be expected here. Eventhough the NEEs representing the different modes of propagation are found to possess soliton solutions, but the interaction between these modes to the lowest order of perturbation represents a linear hyperbolic system. It is shown that there exists a soliton -wave interaction, causing the decay of solitons.

Nonlinear excitations in low dimensional magnetic systems have generated a great deal of theoretical and experimental interest during the recent years. The propagation of electromagnetic waves through a ferromagnet is important in connection with the behaviour of ferrite devices at microwave frequencies such as ferrite loaded wave guides. The propagation of electromagnetic waves in a ferromagnet obeys nonlinear equations with dispersion and dissipation. The present study deals with the propagation of weak nonlinear waves with long wavelength through a ferromagnetic medium with damping. By using the reductive perturbation method the system of equations has been reduced to an integro-differential equation in (2+1) dimensions. We have analyzed the solution of this equation for two different cases. In the dissipation dominant case, the equation exhibits shock wave solution. But in the dispersion dominant case the equation reduced to the modified form of Kadomtsev-Petviashvili equation and the solitary wave solution of this equation is also found out. The energy and the momentum expressions for this equation are also determined.

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Contents

Chapter 1

Nonlinear Science

If everything were linear,
nothing influences nothing.
A. Einstein.

1.1 Introduction

Nonlinear science is a science of evolution and complexity [1]. Nonlinear science introduces a new way of thinking and a new way of looking at natural systems and uses a large variety of methods from various disciplines but through the process of continual switching between different views of the same reality, these methods are cross -fertilized and blended into a unique combination that gives them a marked added value. Most important of all, nonlinear science helps us to identifying the appropriate level of description in which unification and universality can be expected [2]. The fundamental laws of mass- action are inadequate for understanding or even formulating the complexity induced by the evolution of nonlinear systems. In contrast, solitons, attractors, fractals and multifractals, normal forms, Lyapunov exponents, entropies, correlatión functions, etc. are parts of the new scientific vocabulary proposed by modern nonlinear science and provide a pragmatic way to meet a challenge where classical approaches fail. Nonlinear behavior is deeply rooted in the fundamental laws of classical mechanics. Earlier it was believed that nonlinearity is insignificant in physical problems [3].

A linear system is a system in which the observed effects are linked to the underlying causes by a set of laws reducing for all practical purposes to a simple proportionality is shown to provide, at best, only a partial view of the natural world. In many instances and in our day to-day experiences, we come across qualitative deviation from the regime of proportionality. Physical phenomena concern the inter-relationship of a set of physical variables (cause and effect) which are deterministic (with some accuracy). Nonlinear phenomena involve those sets of variables such that an individual change of one variable does not provide a proportional change in the resultant variable. In the case of linear systems the principle of superposition holds, that is, the ultimate effect of the combined action of two different causes is merely the superposition of the effects of each cause taken as individually. But in the case of nonlinear systems the superposition principle breaks down, because adding two elementary actions can induce dramatic new effects reflecting the onset of co-operativity among the constituent elements [4]. This can give rise to unexpected structures whose properties can be quite different from those of the underlying elementary laws, in the form of abrupt transition, a multiplicity of states, pattern formation or an irregular unpredictable evolution in space and time referred to as deterministic chaos [5].

1.2 Nonlinear evolution equations

Nonlinear physics has invaded many research disciplines over the recent years and such interdisciplinary activity is documented by an ever increasing rate of scientific work.

This is not too surprising, since most phenomena of the macroscopic inanimate and living world, including the complex behavior of the brain, are of nonlinear nature. On a long term basis, the ultimate role of physics in exploring complex nonlinear systems will be to help unravel the mechanisms behind it and to develop a deeper qualitative understanding in terms of fundamental laws and equations of motion. Such expectations require, besides the direct application of established methods to new systems, a conceptual progress that goes beyond the current state of the field. In the strife for a better understanding of such complicated physical phenomena such as weather changes, earthquakes, or cardiac arrythmias, scientists have found it useful to introduce mathematical models *whose time evolution* might exhibit some features very similar to those of the original phenomena. These models are very often systems of *differential equations*, with n dependent variables (or n unknowns) which are in general functions of the independent time variable t, and perhaps of the space variables x, y, z also [6].

Nonlinear dynamical systems are of two types. Integrable systems and non-integrable systems. Most of the non-integrable systems exhibit chaotic behaviour. That is most dynamical systems do not follow simple, regular and predictable patterns but run along a seemingly random, yet well-defined trajectory in phase-space. The generally accepted name for this phenomenon is chaos. Integrable systems possess regular behaviour. Among the nonlinear partial differential equations (PDEs) of physical interest, an important role is played by the class of evolution equations which possess remarkable features such as solitary wave solutions [7] which preserve their shape and speed after interaction. The nonlinearity inherent in most classical equations of motion makes the question of stability and the prediction of long term behaviour all the more interesting.

Determination of the dynamical properties of physical systems with competition between discreteness, nonlinearity and dispersion has recently attracted a growing interest due to the wide applicability of such systems The balance between nonlinearity and dispersion provides the existence of soliton-like solutions [8]. Due to their robust character the solitonic excitations find applications in various branches of physics such as plasma physics, optical fibre communication, cosmology, condensed matter physics, etc.[9-16]

1.3 Solitary waves

The theory of solitary waves is related to many areas of mathematics and has several applications to physical sciences. Solitary waves are solutions of certain class of non-linear partial differential equations with the characteristic feature that the solutions represent wave pulses that remain unchanged in amplitude and speed when they evolve in time and space. Actually this property is that of an ideal wave pulse which is not distorted by any physical phenomenon like wave dispersion or damped by dissipative forces. But in reality these conditions are never met with as such. At the same time it has been observed that such an "ideal wave pulse " can exist under certain circumstances where the effect of some physical phenomena may contribute to the origin of "solitary waves It was such an ideal wave pulse that Sir John Scott Russel [17]observed in a Scottish canal in 1834. It was the first ever observation of a solitary wave and quite accidental in the sense that such a phenomenon was not even imagined by anybody, prior to this observation. What Sir John Scott Russel observed was a *well defined heap of water* .....continued on its course along the channel apparently without change of form or diminution of speed.

Solitary waves made their first appearance in the world of science with the beautiful report presented by Russel. According to him the propagation of an isolated wave, was a consequence of the properties of medium rather than the circumstances of the waves generation. Some nonlinear differential equations admit solutions consisting of isolated waves that can propagate and undergo collisions without losing their identity. Subsequent, careful observations revealed that a wave of water is a nonlinear phenomenon because its speed depends on its frequency. The long lived solitary wave of water observed by Russel was one in which the nonlinear effect of the speed of height balanced the nonlinear dependence of its speed on wavelength. At the time of the observation of this phenomenon there was no mathematical explanation or an equation of water waves in shallow water. It remained so until Korteweg and deVries (KdV)[18] developed an equation in 1895, to explain the formation of waves in shallow water. This equation now known as KdV equation possesses solitary wave solutions.

The non-linear evolution equation governing long, one dimensional, small amplitude surface gravity waves propagating in a shallow channel of water derived by them is given by:

$$\frac{\partial\eta}{\partial\tau} = \frac{3}{2}\sqrt{\frac{g}{h}}\frac{\partial}{\partial\xi}(\frac{1}{2}\eta^2 + \frac{2}{3}\alpha\eta + \frac{1}{3}\sigma\frac{\partial^2\eta}{\partial\xi^2}) \tag{1.1}$$

where $\sigma = \frac{1}{3}h^3 - \frac{Th}{\rho g}$. Here η is the surface elevation of the wave above the equilibrium level h. α is a small arbitrary constant related to the uniform motion of the liquid, T is the surface tension and ρ is the density. The terms 'long' and 'small' are meant in comparison with the depth of the channel. This equation may be brought into dimensionless form using the following transformations.

$$t = \frac{1}{2}\sqrt{\frac{g}{h\sigma}}T, \quad x = -\sigma^{-1/2}\xi \tag{1.2}$$

$$u = \frac{1}{2}\eta + \frac{1}{3}\alpha$$

Hence we obtain

$$u_t + 6uu_x + u_{xxx} = 0 \tag{1.3}$$

which is the well known KdV equation which finds applications in a number of physical systems such as plasma dynamics, stratified internal waves, ion-acoustic wavesand lattice dynamics, etc. This equation possess a solitary wave solution: where k and x_0 are constants. The velocity of this wave $v_p = 4k^2$ is proportional to the amplitude $A = 2k^2$

$$u(x,t) = 2k^2 sech^2 \left[k(x - 4k^2 t - x_0)\right] \tag{1.4}$$

In 1965 Zabusky and Kruskal[19-20] introduced the word 'soliton' to characterize waves that do not disperse and preserve their form during propagation and after collision. Zabusky and Kruskal discovered using computer simulation that the solitary wave solutions of KdV equation retained their forms and speeds and were stable even if they collided with one another. At the heart of these observations is the discovery that these nonlinear waves can interact strongly and then continue thereafter almost as if there had been no interaction at all (except for a small phase change). This behaviour is similar to that of particles in elastic collisions with each other Observing this particle -like behaviour of solitary waves, Zabusky and Kruskal coined the word 'solitons' to describe them. Such solitons may be observed in shallow water where a taller wave moving with a greater speed overtakes a shorter wave without any change of initial shape or speed occurring to either of the waves due to the 'mixing' of the two during their 'collision' The word solitary wave is usually reserved for single soliton solution or solitons in isolation, say, at infinite separation [21-23]. The soliton phenomena and integrable nonlinear equations represent an important and well established field of modern physics, mathematical physics and applied mathematics. Solitons are found in a wide range of disciplines in physics like hydrodynamics, plasma physics, nonlinear optics, solid state physics, etc. Soliton solutions are a beautiful mathematical construct based on the integrability property of the nonlinear evolution equation. They are the elementary excitations in nonlinear dispersive systems.

The theory of partial differential equations have an important role to play in the study of nonlinear wave propagation. Such equations frequently arise as a result of the determination of the asymptotic behaviour of either a higher order equation or a complicated system of equations, and they are often called far-field equations. The existence of solitary wave solutions implies perfect balance between nonlinearity and dispersion which usually requires rather specific conditions and cannot be established in general. The following mathematical techniques namely Painleve analysis [24], Lax method [25], Inverse Scattering Transform method (IST) [20], Hirota's bilinearization [27] and Backlund transformation [26-28] are used for identifying integrable systems.

One of the main uses of integrability lies in the fact that it allows us to obtain global information on the long time behaviour of the system usually through the existence of conserved quantities which remain constant in value through out the time evolution of the system.

1.4 Painleve Analysis

Over the years, various methods have been developed for studying wave propagation through nonlinear media. To begin with, it is desirable to have a simple approach for deciding whether a given dynamical system is integrable or not. Integrable, or more accurately exactly solvable equations have wide applications in theoretical and mathematical physics. Nonlinear integrable systems were discovered as early as the eighteenth century. Historically, the first application of these ideas is due to Kovalewskaya [29] in her work on the rigid body problem.

Her approach was mainly to determine the conditions under which the only movable singularities, i.e., those singularities whose positions are initial-condition -dependent, are ordinary poles. Kovalewskaya observed that the majority of non-integrable systems could be integrated in terms of elliptic and consequently meromorphic functions and thus cannot have any movable critical points. Kovalewskaya's idea was pursued further by Painleve. The method of verifying the integrability of equations through an analysis of the arrangement of critical points of their solutions in the complex plane is called the Painleve test. An ordinary differential equation in the complex domain is said to be of Painleve type (or has the Painleve property) if the only movable singularities of all its solutions are poles. This means that there will be no more essential singularities. Now various authors have applied the connection between the Painleve property and integrability. The Painleve method can be applied to systems of ordinary and partial differential equations alike.

The basic idea is to expand each dependent variable in the system of equations as a Laurent series about a pole manifold. If the equations are a set of ordinary differential equations (ODEs), then this simply means that one looks for solutions as Laurent series in the complex time variable $(t - t_0)$. In order that the equations for the coefficients of the various powers of $(t - t_0)$ in the Laurent expansion have self-consistent solutions, certain conditions on the structure of the given system of equations are required. If the system satisfies these conditions, and the number of undetermined constants appearing in the Laurent series together with t_0 is equal to the order of the system of equations, then we say that the solution has the Painleve property. This conjecture has proved to be quite valuable, although there are clearly certain drawbacks such as the difficulty of identifying all possible reductions of the PDEs without any need for reductions.

Weiss, Tabor and Carnevale [30-31] introduced the Painleve property for PDEs. A PDE is said to possess the Painleve property, if the solutions of the PDE are 'single-valued' in the neighborhood of non-characteristic, movable singularity manifolds. Weiss,Tabor and Carnevale proposed a method of applying the Painleve PDE test to a given PDE which is analogous to the algorithm given by Ablowitz, Ramani and Segur [32-34] to determine whether a given ODE might be Painleve type or not. The novel feature of the WTC approach is the flexibility contained in the singular manifold function. It is this flexibility that allows the Laurent expansion to be collapsed into Backlund transformation which gives rise to the Lax pairs[25]. The ultimate goal is to show that the generalized Laurent expansion can not only show that a system is integrable, but also that the expansion can be used to provide an algorithm which successfully captures all its properties; namely, the Lax pairs, the Backlund transformations, the Hirota equations, the motion invariance, symmetries and commuting flows, the geometrical structure of the phase space and finally the algebraic properties (symmetries)[35] which make the system's exact integrability transparent and possible.

1.5 Inverse Scattering Transform

Many physical problems are modeled by nonlinear partial differential equations. There was no unified method by which the nonlinear PDEs could be solved, and the solitons were often obtained by *ad-hoc* methods. As is well known, the basic mathematical method to analyze any kind of wave propagation in linear media is the famous Fourier transform method. The Fourier transform method can not be used for nonlinear systems where the superposition principle does not exist. However, one may try to invent another kind of (nonlinear) decomposition to obtain *nonlinear modes* whose evolution will be reduced to trivial oscillations similar to that given by the Fourier transform. Such a decomposition is known to exist only for some special nonlinear equations (the so-called exactly integrable equations), and it is called *the inverse scattering trans-*

form method [24]. In some sense, the inverse scattering transform provides an inherent similarity between the modes of linear inhomogeneous systems and those of nonlinear inhomogeneous systems.

The main idea of the nonlinear decomposition based on the inverse scattering transform is to find an appropriate linear eigenvalue problem which includes the input beam u(t=0,x) as an effective of the potential The main property of this linear eigenvalue problem is, in spite of an explicit dependence of the potential on t, the eigenvalues are conserved quantities provided the wave field $u(t, x)$ satisfies the primary nonlinear equation. Finding the eigenvalue problem with this property implies that evolution equation is a common feature of the integrability technique based on the inverse scattering transform method. The property that the eigenvalues remain invariant under time means that if they are found (or known) for $u(t = 0, x)$, they remain the same for any t. For a given $u(t = 0, x)$ the eigenvalue problem can be solved as a scattering problem for an auxiliary eigenfunction $\psi(x; \lambda)$, λ being the spectral problem eigenvalue. The scattering data can be obtained from the amplitude a(λ) of a transmitted wave, the amplitude b(λ) of a reflected wave, the eigenvalues of the discrete spectrum $\{\lambda_n\}$,and the normalized coefficients b_n of the eigenfunctions of the discrete spectrum. Similar to the machinery of the standard Fourier transform method, the evolution of the scattering data $\{a(\lambda), b(\lambda), \lambda_n, b_n\}$ is trivial, and the solution of the primary nonlinear equation is then found using results of the inverse scattering method. Each eigenvalue of the discrete spectrum of the scattering problem governs a localized solution, i.e., soliton. Therefore, the stationary nature of the eigenvalues provides the important property of the stability of the soliton beams when undergoing collisions. Hence, solitons are important not only as particular solutions of the nonlinear equation, but as unique solutions whose stability is guaranteed by the invariant property of the corresponding eigen value problem.

The most significant result was the development by Gardner, Green, Kruskal and Miura (GGKM) [36-37] of a method for the exact solution of the initial-value problem for the KdV equation; the initial values decay sufficiently rapidly through a series of linear equations. The aim is to solve the KdV equation for (x, t) $x \in \Re$, $t \to 0$ subject to the initial condition

$$u(x, 0) = f(x)$$

where $f(x)$ decays sufficiently rapidly as $| x | \to \infty$. The basic idea is to relate the KdV equation to the time-independent Schroedinger scattering problem,

$$L\psi := \psi_{xx} + u(x, t)\psi = \lambda\psi \tag{1.5}$$

which has been extensively studied by both mathematicians and physicists. The motivation for this equation came from studying the Miura transformation relating solutions of the KdV and modified KdV equations. If $U(x, t)$ is a solution of the mKdV equation

$$U_t - 6U^2U_x + U_{xxx} = 0 \tag{1.6}$$

then

$$u = (U^2 + U_x) \tag{1.7}$$

is a solution of the KdV equation

$$u_t - 6uu_x + u_{xxx} = 0 \tag{1.8}$$

Eq.(1.7) may be viewed as a Riccati equation for u in terms of U. It may be linearized by the transformation $U = \frac{\psi_x}{\psi}$ and yields:

$$\psi_{xxx} - u\psi = 0. \tag{1.9}$$

Since KdV equation is Galilean invariant, that is invariant under the transformation $(x, t, u(x,t) \rightarrow (x - ct, t, u(x,t) + \frac{1}{6}c)$ where c is some constant, t plays the role of a parameter and $U(x,t)$, the potential. Therefore it is natural to consider the time-independent Schrodinger equation

$$\psi_{xxx} + u(x,t)\psi = \lambda\psi \tag{1.10}$$

rather than equation (1.9). The direct Scattering problem is to map the potential into the scattering data. The inverse scattering problem is to reconstruct the potential from the scattering data. The time dependence of the eigenfuctions of (1.5) is given by

$$\psi_t = (\gamma + u_x)\psi - (4\lambda + 2u)\psi_x, \tag{1.11}$$

where γ is an arbitrary constant. Assuming that λ_t=0, then from (1.5) and (1.11) we obtain

$$\psi_{txx} = [(\gamma + u_x)(\lambda - u) + u_{xxx} + 6uu_x]\psi - (4\lambda + 2u)(\lambda - u)\psi_x \tag{1.12}$$

$$\psi_{xxt} = [(\gamma + u_x)(\lambda - u) - u_t]\psi - (4\lambda + 2u)(\lambda - u)\psi_x \tag{1.13}$$

Therefore (1.12) and (1.13) are compatible (i.e.,$\psi_{xxt} = \psi_{txx}$), if and only if, u satisfies the KdV equation

$$u_t + 6uu_x + u_{xxx} = 0 \tag{1.14}$$

The solution of (1.14) corresponding to $u \to 0$ as $|x| \to \infty$ proceeds as follows.

At time $t = 0$, given $u(x, 0)$ we can solve the direct scattering problem.

$$\psi_{xxx} + u(x,t)\psi = \lambda\psi \tag{1.15}$$

The spectrum of the Schroedinger equation consists of a finite number of discrete eigenvalues $\lambda = k_n^2, n = 1, 2, ...N$ for $\lambda > 0$ and a continuum $\lambda = -k^2$ for $\lambda < 0$. The eigenfunctions corresponding to these eigenvalues may be computed and their asymptotic behaviour can be written as follows: for $0 < \lambda = k_n^2$,

$$\psi_{n(x,t)} \sim C_n(t)exp(\pm k_n x) \ \ as \ \ x \to \pm\infty \tag{1.16}$$

with

$$\int_{-\infty}^{\infty} \psi_n^2 dx = 1.$$

and for $0 > \lambda = -k^2$

$$\psi(x,t) \sim e^{-ikx} + \gamma(k,t)e^{ikx} \ \ as \ \ x \to -\infty \tag{1.17}$$

$$u(x,t) \sim a(k,t)e^{-ikx} \ \ as \ \ x \to \infty \tag{1.18}$$

where $\gamma(k,t)$ is the *reflection coefficient* and $a(k,t)$ is the *transmission coefficient.* Hence we have the scattering data at time $t = 0$:

$$S(\lambda, 0) = (\{k_n, C_n(0)\}_{n=1}^{N} r(k, 0), a(k, 0)) \tag{1.19}$$

Time evolution of the scattering data is

$$C_n(t) = C_n(0) exp(4k_n^3 t)$$
$$a(k, t) = a(k, 0)$$
$$r(k, t) = r(k, 0) exp(8ik^3 t) \tag{1.20}$$

where k_n, a constant, and $n = 1, 2, ...N$.

The scattering data at time t is then given by

$$S(\lambda, t) = (\{k_n, C_n(t)\}_{n=1}^{N}, r(k, t), a(k, t)) \tag{1.21}$$

The inverse scattering problem is to reconstruct the potential $u(x, t)$ from a knowledge of the scattering data $S(\lambda, t)$, which is the required solution of the KdV equation. This problem was considered by Gel'fand and Levitan [38]. Using the scattering data we define a function F(x;t) as

$$F(x; t) = \sum_{n=1}^{N} C_n^2 exp(-k_n x) + \frac{1}{2\pi} \int_{-\infty}^{\infty} r(k, t) e^{ikx} dk \tag{1.22}$$

Then solving the linear integral equation called Gel'fund -Levitan-Marchenco equation:

$$K(x, y; t) + F(x + y; t) + \int_{x}^{\infty} K(x, z; t) F(x + y; t) dz = 0 \tag{1.23}$$

The potential can be reconstructed by using the equation

$$u(x,t) = 2\frac{\partial}{\partial x}[K(x,x;t)] \tag{1.24}$$

This method is therefore conceptually analogous in many ways to the Fourier transform method for solving linear equations. The only difference is that the final step is highly nontrivial. If the reflection coefficient is zero $r(k,0) = r(k,t) = 0$, we obtain the special soliton solution of the KdV equation. In this case

$$F(x;t) = \Sigma_{n=1}^{N} C_n^2(t)\exp(-k_n x) \tag{1.25}$$

Substituting this into the GLM equation, we finally obtain the N-soliton solution for the KdV equation. The N-Soliton solution have the form as t$\rightarrow \pm\infty$.

$$u_n(x,t) \sim 2k_n^2 \sec h^2 k_n(x - 4k_n^2 t + x_n) \tag{1.26}$$

where x_n is constant, $n = 1, 2, \ldots.. N$ There is a one-to-one relationship between the number of discrete eigenvalues and the number of solitons which emerge asymptotically. As mentioned above, these waves interact in such a way as to preserve their identities in the asymptotic limit. However by using the inverse scattering method to solve the KdV equation, we are able to mathematically confirm the numerical observations of Zabusky and Kruskal. The number of discrete eigenvalues is equal to the number of solitons.

1.6 Wave propagation through nonlinear dispersive media

The propagation of waves through dispersive media is of great interest since nonlinear dispersive systems are abundant in nature. Most of the wave motions are represented by hyperbolic partial differential equations. In the time dependent situations the partial differential equations mostly associated with propagation are of hyperbolic type, and they may be either linear or nonlinear.

Nonlinearity in waves manifests itself in a variety of ways[39]. In the case of waves governed by hyperbolic equations, the most striking aspect is the evolution of discontinuous solutions from arbitrary well behaved initial data. In the case of other types of equations, the effect of nonlinearity is usually tampered by the effects of dispersion and dissipation that might also be present. In general, when dispersive effects are weak, long wavelength behaviour is possible. In the other extreme of strong dissipation, a highly oscillatory behaviour occurs, though the envelope of such oscillations often exhibits some of the characteristics of long waves. Apart from leading to the decay of a disturbance, dissipation also exerts a smoothing effect and like dispersion, can be combined with the effects of nonlinearity to make travelling (progressing) waves possible. In order to make precise the notions of dispersion and dissipation, we consider the simplest case of a linear operator L and the equation $L(u) = 0$ for the function $u(x,t)$ in one space dimension x and time t. Then, using complex notation, any harmonic one dimensional plane wave $\tilde{u}(x,t)$ of amplitude A and wave number m moving with speed c may be written in the form

$$\tilde{u}(x,t) = Re\left[Aexp(im(x-ct))\right] \tag{1.27}$$

$$= Re\left[Aexp(i(kx - \omega t))\right] \tag{1.28}$$

We seek plane wave solution of this form which satisfies $L(u) = 0$. Substituting $\tilde{u}$ into this equation, we arrive at a compatibility condition $D(\omega, k) = 0$ to be satisfied by the wave number k and the angular frequency ω. This compatibility condition is known as the dispersion relation for $L(u) = 0$ and it is often solved for ω in terms of k to give an alternative form of the dispersion relation

$$\omega = \omega(k) \tag{1.29}$$

For each real k, the allowed values of ω are obtained by solving the dispersion relation $D(\omega, k) = 0$. Each solution is called a normal mode. The dispersion relation in general gives complex ω for real k. The phase velocity v_p of the plane wave is defined as $v_p = \frac{\omega}{k}$. The real part, $Re(v_p)$, is the speed of propagation of geometrical features of the wave, while the group velocity $v_g = \frac{\partial \omega}{\partial k}$ relates to the speed of propagation of the energy of the wave. If $Im(\omega) < 0$, the waves will decay exponentially with time and this attenuation process is called dissipation. Conversely, if $Im(\omega) > 0$, the waves will grow exponentially with time and this growth process is called instability. If $\frac{\partial v_p}{\partial k} \neq 0$, neither dissipation nor instability will arise. This is the case of pure dispersion.

Nonlinear waves are more complicated because in general, the frequency ω depends not only on the wave number but also on the amplitude. Various asymptotic methods have been developed for nonlinear dispersive wave problems. The fundamental ideas underlying these methods are as follows. Let us first consider the asymptotic behaviour of linear dispersive waves. This can be constructed by means of superposition of elementary solutions in the form of sinusoidal progressive wave trains, that is

$$u(x,t) = Ae^{i(kx-\omega t)} + c.c \tag{1.30}$$

There are two approaches to the formal derivations of asymptotic equations, the differential approach and the Fourier approach. In the case of Fourier approach the wave propagation in a dispersive media may be represented by means of a Fourier representation in terms of the spectrum function at some given time. And in order to derive asymptotic equations for nonlinear dispersive waves it is necessary to devise some form of nonlinear perturbation scheme. There are a number of nonlinear perturbation schemes which are of mathematical interest. The important methods among them are the reductive perturbation method(RPM) proposed by Taniuti *et al* [42-46], the Krylov-Bogoliubov-Mitropolsky [41] (KBM), multiple scale expansion method [39] and the derivative expansion method [39]. All these methods can be applied for studying the asymptotic behaviour of various physical systems both for the nonlinear wave modulation case and for the long wavelength approximation of the nonlinear dispersive waves.

In this thesis, we have studied the salient features of wave propagation through different nonlinear dispersive media using reductive perturbation method and K-expansion method. A study of different aspects of wave propagation through nonlinear media is of considerable importance from both theoretical and experimental points of view. The study has very much importance in many physical situations such as ion-acoustic waves and Alfven -like disturbances in plasmas, propagation of phonons in crystal lattices and electromagnetic waves in optical fibres. To study the asymptotic behaviour of nonlinear dispersive waves, Gardner and Morikawa [36] introduced the scale transformations derived from the linearized asymptotic behaviour of long waves and they combined this transformation with a perturbation expansion of the dependent variables so as to describe nonlinear asymptotic behaviour This type of perturbation has been developed and formulated in a general way and is called the reductive perturbation method by Taniuti and collaborators [42-46]. This method is a systematic way for the reduction of a fairly general nonlinear system to a single tractable nonlinear equation describing the far-field behaviour and is known as the Reductive Perturbation Method (RPM).

1.7 Reductive Perturbation Method (RPM)

The evolution equation describing the wave propagtion through nonlinear media contain in general several dependent variables and it is not easy to solve them and we need a procedure which, in a systematic way, will reduce such sets of equations to simpler forms. The natural choice of such a procedure will be a perturbative one and hence the procedure of reductive perturbation theory is usually used. One of the useful features of this form of perturbationtheory is that it enables us to look in a natural way for long waves, ie, waves whose wavelengths are long compared to a typical length scale. On the mathematical level, in order to build this length scale into the original equations of motion, we need to rescale both space and time of long wavelength phenomena. This rescaling enables us to isolate from the system of equations, the relevant equations of motion which describe how the systemreacts on the new space and time scales.

$$\xi = \varepsilon^{\alpha}(x - \lambda t)$$
$$\tau = \varepsilon^{\beta} t \tag{1.31}$$

The new variables ξ and τ are long in the sense that it needs a large change in x and t in order to change ξ and τ appreciably. In order to determine the values of α and β, a plausibility argument is needed. When the basic equations are powers of ε and space and time are also expanded as in Eq.(1.31), then a suitable choice for the values of α and β are often apparent. Intuitively it can be seen that if α and β are chosen to be large many of the dependent variables would appear to be independent and in order to obtain evolution equations it would be necessary to consider higher order terms in the perturbation expansions and hence α and β are given small integral or half integral values.

This method has been developed and formulated in a general way, as what is later called the RPM, by Taniuti and his collaborators (Taniuti and Wei), (Taniuti and Washimi),(Taniuti and Yajima). This method have introduced a systematic way for reducing a system of equations to a single tractable equation describing the long term behaviour of the physical system. The RPM was first established for the long-wave approximation (Taniuti and Wei, Washimi and Tanuiti, Kakutani) and then for wave modulation problems [39]. The long-wave approximation of the RPM is the basic mechanism we shall be concerned particularly for carrying out this work.

1.8 Stability analysis of the solitary wave solution

Only a few systems of practical interests are integrable The state of the system can be viewed as a point in phase space; and the evolution in a small but finite time maps this point to another point, unless singularities intervene. These phase spaces can be viewed as an alternative form of dynamics (discrete time), and one could ask questions about long term behaviour and the qualitative aspects of wave propagation. Therefore the question of stability and the secular behaviour are of great importance. These questions are expected to be complicated for nonlinear systems, but they can become nontrivial even for innocent looking systems. Just because the equations of motion of a dynamical system have been integrated it does not follow that the system behaves in a satisfactory manner. Slight disturbances can dramatically alter orbits and the conserved quantities of dynamics vary irregularly over the phase space. Long term trends cannot be predicted on ideas abstracted from simple dynamical systems [60].

For application to the solitary -wave problems the definition of stability needs care, particularly as there is a sense in which no solution of KdV can be stable over an unbounded time interval. A device entailing the definition of a certain quotient space is used to discriminate the stability of solitary waves in respect of shape which is a more reasonable property to investigate than the conditions for absolute stability.

If the system is driven or pumped with energy through some mechanism, for example, a rotation, a background flow or a heat gradient, then potential energy is made available to the waves. There may be a 'control parameter' within the mathematical model, whose role can be important in that the system may become unstable under the influence of the background energy flow when the parameter passes through a critical value. The infinitesimally small disturbances will grow if the control parameter is in the post-critical region. Rowlands initiated and with Infield, [60] developed a scheme based on the assumption that a nonlinear wave undergoes a long wavelength perturbation. Thus, if the wave vector of the perturbation is K, we assume $K \ll k$. We should assume $\frac{\Omega}{\omega} \sim \frac{K}{k}$ where K and Ω are the wave vector and frequency of the perturbation, while k and ω describe the basic nonlinear wave structure. The first step is to linearize the equation around the stationary wave $\phi(x)$. We take perturbed quantities to be of the form $P(x)e^{i(Kx-\Omega t)}$ where $P(x)$ is periodic with the same wavelength λ as ϕ, and remove terms secular in x in each order in K/k. Integrations over $\lambda = \frac{2\pi}{K}$ of the nonlinear wave are involved and equations of the general form

$$G(K, \Omega, A) = 0 \tag{1.32}$$

are obtained as consistency conditions. This usually occurs in the second order (K^2) calculation. The K-expansion method was given a mathematical basis by Rowlands in

1969 and applied to the problem of instabilities generated by two counter streaming electron beams in a hydrogen plasma when a nonlinear electrostatic wave is present. The next chapter deals with the study of electromagnetic wave propagation through cold collisionless plasma using the reductive perturbation method.

Chapter 2

Reductive perturbation analysis of electromagnetic wave propagation through a cold collisionless plasma

If nature were not beautiful,
it would not be worth knowing,
and if nature were not worth knowing,
life would not be worth living.

H.Poincare

2.1 Introduction

Plasmas are very unique media, which are able to sustain various types of oscillations. In particular under the effect of external magnetic fields, a number of electromagnetic waves in a plasma exhibit a full variety of peculiarities in their propagation properties. Amplitudes of these waves are of such a large level that nonlinear effects prevail in competition with dispersion properties of the waves. One of the most important aspects of present day plasma research is the emergence of nonlinear solitonic excitations. This nonlinear phenomenon is one of the fundamental mechanism for the energy and momentum transfers between waves and particles in the weak plasma turbulence. The knowledge of the behavior of electromagnetic waves in plasma is rather essential for understanding the salient features of Alfven like disturbances, which have been frequently observed in the solar wind as well as in the Earth's ionosphere and magnetosphere. In the solar wind, large amplitude incompressible magnetic field perturbations have been observed. The existence of these perturbations can not be explained by exact magnetohydrodynamic theory. It has been found that the stationary solutions of the nonlinear equations can be represented in the form of vortical structures. The latter can be readily associated with the coherent nonlinear structures that have been experimentally observed in the Earth's magnetosphere. As a matter of fact theoretical understanding of the nonlinear wave phenomena in plasma and other dispersive media has achieved remarkable success [61,64].

The propagation of solitary waves in a plasma is of much current interest. Plasma allows for two types of waves, *transverse electromagnetic waves* and *longitudinal electrostatic waves.* The latter are nothing else but oscillations of the electrostatic potential and are not accompanied by magnetic fluctuations. The electrostatic modes are confined to the plasma, because oscillations of the electrostatic potential can be maintained only inside the plasma boundaries [65].

It is well known that electron-positron plasmas are frequently found in the early universe in pulsars' magnetosphere and in active galactic nuclei as well as in other astrophysical objects [67-68]. The study of wave phenomena is rather crucial with regard to the understanding of disturbances in such plasmas. Thus linear and nonlinear electrostatic and electromagnetic waves have been investigated by several authors in an electron-positron plasma without and with ions. Specially, the possibility of electromagnetic solitons in the presence of a stationary ion background in an electron -positron plasma has been pointed out.

However, in the early universe and near the polar caps of pulsars, there might exist localized structures in addition to sufficiently cold electron and positrons. Therefore it is highly desirable to understand the properties of wave phenomena in such a multi-species electron-positron plasma [69].

Soliton solutions in hydromagnetic plasma models are usually based on the concept of plasma as a medium with a special type of nonlinear dispersion .The linear evolution is most appropriately described in terms of Fourier modes, which have a constant amplitude and a well defined group velocity (The speed with which a wave packet peaked at that Fourier mode would move in ordinary space). To identify the nonlinear effects, we consider a specific Fourier mode and the frame of reference which moves with the corresponding group velocity. Recent concerns with soliton phenomena are to study whether the notion of soliton is effective even for non-integrable systems, in other words, if solitons play a similar role in real systems as they do in integrable systems [35]. In this context there have been reports on soliton resonance in multi-dimensional systems, the onset of stochasticity in soliton systems under the action of external forces and so on. It is known that various perturbation methods, for instance the Krylov-Bogolibov-Mitropolsky [51] method as applied by Kakutani and Sugimoto, Buti and Durrani or the reductive perturbation method developed by Taniuti and his collaborators [48-58] can be used to describe the long-time behaviour of the problem. To evaluate the weak nonlinear effects it is convenient to consider a specific Fourier mode and follow it by going over to a stretched frame of reference. The problem of nonlinear electron oscillations in a cold plasma is of considerable importance. Taniuti and Washimi [42] were the first to use reductive perturbation method to study the propagation of a slow modulation of a quasimonochromatic waves through plasma. This method has been used later to study a wide class of nonlinear wave systems [52-56].

In this chapter we have studied the propagation of electromagnetic waves through a cold collision free plasma using the reductive perturbation method (RPM) in (1+1) dimensions and in (2+1) dimensions. It is found that to the lowest order of perturbation, the system of equations can be reduced to modified Kortewegde-Vries equation [18] in the case of (1+1) dimensions and to modified Kadomtsev-Petviashvili equation [66] in (2+1) dimensions. Here plasma is treated by the simplest of models namely an electron fluid.

2.2 Mathematical Formulation of the problem

A plasma consists of an assembly of charged particles, interacting with each other through the Coulomb force. The movement of each plasma particle is governed by the local electric field; at the same time, the particle is also a source of electric field. In order to see what happens in various physical situations, we shall need to obtain solutions by simultaneously solving the equation of motion and Maxwell's equations. This is known as the requirement of self-consistency. A transformation of the nonlinear self-consistent equations for a collisionless cold plasma with stationery ions makes it possible to obtain new classes of analytic solutions. Since it has been discovered by Zabusky and Kruskal [19] that solitary wave plays an important role in nonlinear wave propagation in dispersive media, it seems worthwhile to investigate the existence and properties of solitary waves in various cases of collision-free plasma. By cold collision-free plasma means that there is no random thermal excitation for the electrons and ions in the plasma, and that the electrons and ions are initially motionless and the pressure term in the equations of motion becomes zero. In the present analysis we are taking a stationary background of cold ions. The electron fluid model for plasma is not only of academic interest but also applicable to realistic plasmas such as the ones produced by the electric discharge and the electron cyclotron heating or resulting from the micro-instabilities leading to the electron heating [64-65]. We shall consider nonlinear plane waves with constant profile propagating in a cold collision-free magnetized plasma. The basic equations relevant to the present problem are the following [69]

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \vec{v}) = 0 \tag{2.1}$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}.\nabla)\vec{v} = -\frac{e}{m}[\vec{E} + \frac{\vec{v} \times \vec{B}}{c}] \tag{2.2}$$

$$\nabla \times \vec{B} = \frac{1}{c}[\frac{\partial \vec{E}}{\partial t} + 4\pi \vec{j}] \tag{2.3}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{2.4}$$

$$\nabla.\vec{E} = \frac{4\pi e}{m}(\rho - \rho_0) \tag{2.5}$$

where ρ the electron density, v the electron velocity, ρ_0 the static uniform background density of cold ions. E and B are the electric and magnetic fields respectively. Here we consider an equilibrium state defined by $\rho = \rho_0$, $v = 0, E = 0$. For convenience we put $e = m = 1$.

For convenience we introduce a displacement vector field $\vec{S}$ [62] which describes the direction and distance that the plasma has moved away from the equilibrium as

$$\vec{v} = \frac{\partial \vec{S}}{\partial t} + (\vec{v}.\nabla)\vec{S} \tag{2.6}$$

Now substituting Eq.(2.6) in Eqs.(2.2) and (2.3) and simplifying we can write the Eqs.(2.2) and Eq.(2.3) in the form of two coupled partial differential equations

$$\frac{\partial^2 \vec{S}}{\partial t^2} + \frac{\partial}{\partial t}(\vec{v}.\nabla)\vec{S} + [[\frac{\partial S}{\partial t} + (v.\nabla)S].\nabla]\vec{v} = -\frac{e}{m}[\vec{E} + (\frac{\partial}{\partial t}\vec{S}) \times \vec{B} + (\vec{v}.\nabla)\vec{S} \times \vec{B}] \tag{2.7}$$

$$c^2\nabla^2\vec{E} = \frac{\partial^2 \vec{E}}{\partial t^2} + \frac{\partial^2 \vec{S}}{\partial t^2} + \frac{\partial}{\partial t}(\vec{v}.\nabla)\vec{S} \tag{2.8}$$

Assuming that $\vec{B}$ is a linear function of $\vec{S}$ so that we can write

$$(\frac{\partial}{\partial t}\vec{S}) \times \vec{B} = \frac{1}{2}\frac{\partial}{\partial t}(\vec{S} \times \vec{B}) \tag{2.9}$$

$$= \frac{1}{2}\frac{\partial \vec{S}}{\partial t} \times \vec{B} - \frac{1}{2}\frac{\partial \vec{B}}{\partial t} \times S \tag{2.10}$$

Substituting Eqs.(2.10) and (2.4) in Eq.(2.7) we can write

$$\frac{\partial^2 \vec{S}}{\partial t^2} + \frac{\partial}{\partial t}(\vec{v}.\nabla)\vec{S} + [[\frac{\partial \vec{S}}{\partial t} + (\vec{v}.\nabla)\vec{S}].\nabla]\vec{v} = -\frac{e}{m}[\vec{E} + \frac{1}{2}\frac{\partial \vec{S}}{\partial t} \times \vec{B} + \frac{1}{2}\nabla \times \vec{E} \times \vec{S} + (\vec{v}.\nabla)\vec{S} \times \vec{B}] \tag{2.11}$$

Equations (2.8) and (2.11) are a system of coupled nonlinear partial differential equations and from these equations we can study the type of fluctuations that arise when an electromagnetic wave passes through the plasma.

2.3 Reductive perturbation method:

(1+1)dimensional case

Let us now consider a plane wave propagating along the x direction. All the physical quantities are assumed to be functions of one space coordinate x and time t. We will now introduce the stretching variables ξ and τ as described in section 1.7 [39].

$$\xi = \varepsilon(x - Vt) \tag{2.12}$$

$$\tau = \varepsilon^3 t \tag{2.13}$$

where ε is a small parameter measuring the weakness of dispersive effect. The field variables satisfy the following boundary conditions,

$$E_x^i \to 0, \;\; except\, that \;\; E_x^0 \to E_0 cos\phi \tag{2.14}$$

$$E_y^i \to 0, \;\; except\, that \;\; E_y^0 \to E_0 sin\phi \tag{2.15}$$

$$E_z^i \to 0 \tag{2.16}$$

as $\xi \to -\infty, \; i = 0, 1, 2, 3, ...$

$$S_x^i \to 0, \;\; except\, that \;\; S_x^0 \to S_0 cos\phi \tag{2.17}$$

$$S_y^i \to 0, \;\; except\, that \;\; S_y^0 \to S_0 sin\phi \tag{2.18}$$

$$S_z^i \to 0 \tag{2.19}$$

as $\quad \xi \to -\infty$, $i = 0, 1, 2, 3.........$

For an appropriate choice of the coordinate system we can write $S_0 = s = (s_x, s_y, 0) = (s_0 \cos\phi, s_0 \sin\phi, 0)$. It may be rather difficult to solve the coupled equations with general geometrical considerations. Therefore we restrict ourselves to the case in which we are considering the parallel propagation along thedirection of the magnetic field, ie, B_x=constant, $\vec{S} = (S_x, S_y, S_z)$, and $\vec{E} = (E_x, E_y, E_z)$. Let us seek a solution of Eqs.(2.8) and (2.11) under the form of a Fourier expansion in harmonics of the fundamental $\tilde{E}^n \approx expin(kx - \omega t)$

$$E = \sum_{n=-\infty}^{+\infty} E^n \tilde{E}^n \tag{2.20}$$

$$S = \sum_{n=-\infty}^{+\infty} S^n \tilde{E}^n \tag{2.21}$$

Expressing the Fourier components of E and S in powers of the small parameter ε, we can write:

$$S^n(\xi, \tau) = \sum_{j=0}^{\infty} \varepsilon^j S_j^n(\xi, \tau), \tag{2.22}$$

$$E^n(\xi, \tau) = \sum_{j=o}^{\infty} \varepsilon^j E_j^n(\xi, \tau). \tag{2.23}$$

2.4 Dispersion relation

Applying the boundary conditions given by Eqs.(2.14-2.19) and the expansion given by Eqs.(2.20-2.21), Eq.(2.11) can be cast in the following form:

$$[\frac{\partial^2}{\partial t^2} - 2in\omega\frac{\partial}{\partial t} - n^2\omega^2]S_j^n = \frac{\partial}{\partial x} - ink[\sum_{p+q=n} E_j^p \times S_j^q]\frac{1}{2} + E_j^n \tag{2.24}$$

Similarly Eq.(2.8) can be put as:

$$[\frac{\partial^2}{\partial t^2} - 2in\omega\frac{\partial}{\partial t} - n^2\omega^2][E_{jm}^n + S_{jm}^n] = c^2[\frac{\partial^2}{\partial x^2} + 2ink\frac{\partial}{\partial x} - n^2k^2]E_{jm}^n(1 - \delta_{m},x) \tag{2.25}$$

Taking the leading order terms for the order $(j = 0, n)$. of Eqs.(2.24) and (2.25)

$$n^2\omega^2[E_{0,m}^n + S_{1,m}^n] = c^2[n^2k^2E_0^n](1 - \delta_{m,x})$$

$$in\omega S_0^n = \frac{ink}{2}\sum_{p+q=n}(S_0^p \times E_0^q)$$

For $j = 0$ we find that

$$S_0^0 \times E_0^0 = 0$$

Now we can say that E_0^0 is necessarily collinear to S_0^0 and thus define α such that $E_0^0 = \alpha S_0^0$ where E_0^0 and S_0^0 represent the initial static state of the system. Now taking the leading order terms of Eq.(2.25) for the order $(j = 1, n)$

$$n^2\omega^2[E_{1m}^n + S_{1m}^n] = c^2[n^2k^2E_1^n](1 - \delta_{m,x}) \tag{2.26}$$

$$in\omega S^n_{1m} = \frac{ink}{2} S_0 \times [E^n_{1m} - \alpha S^n_{1m}] \tag{2.27}$$

Eq.(2.26) gives the components of S^n_{1m} as functions of E^n_{1m}, $(m = x, y, z)$, we can get

$$S^n_{1x} = -E^n_{1x} \tag{2.28}$$

$$S^n_{1y} = -\gamma E^n_{1y} \tag{2.29}$$

$$S^n_{1z} = -\gamma E^n_{1z} \tag{2.30}$$

where

$$\gamma = (1 - c^2 \frac{k^2}{\omega^2}), \tag{2.31}$$

Substituting for $S^n_{1x}, S^n_{1y}, S^n_{1z}$ in Eq. (2.27), we find a linear homogeneous system for E^n_{1x}, E^n_{1y} and E^n_{1z} It reads

$$n^2\omega^2 E^n_{1x} + \frac{1}{2} ikS_{0y}\beta E^n_{1z} = 0 \tag{2.32}$$

$$n^2\omega^2 \gamma E^n_{1y} + \frac{1}{2} ikS_{0x}\beta E^n_{1z} = 0 \tag{2.33}$$

$$-\frac{1}{2}(1+\alpha) ikS_{0y} E^n_{1x} + \frac{1}{2} ikS_{0x}\beta E^n_{1y} + n^2\omega^2\gamma E^n_{1z} = 0. \tag{2.34}$$

where

$$\beta = (1 + \alpha\gamma) \tag{2.35}$$

This system of equations will have a nontrivial solution only if the determinant of the augmented matrix is zero. The determinant of this system, $\Delta(n)$ is

$$\Delta(n) = n^2\omega^2[-n^4\gamma^2\omega^4 + \frac{k^2}{4}\beta^2(s_x)^2 + \frac{k^2}{4}\gamma\beta(k^2/4)(1+\alpha)(s_y)^2] \tag{2.36}$$

The assumed conditions at infinity are, E_j^n, $S_j^n \to 0$ for $j = 0$ for all 'n' except for $(j, |n|) = (1,1)$ For $n = 1$, $\Delta(1) = 0$ for $j = 1$ and we have the nontrivial solution. But for $n = 2, 3, 4...$, $\Delta n \neq 0$, we have the trivial solution That is for $j = 1$ and $n > 1$, we get $E_1^n = S_1^n = 0$ For $n = 0$, $\Delta(0) = 0$, we can choose $E_1^0 = S_1^0 = 0$ This completes the solution at order $(1, n)$. For the next order, we can proceed in the same manner. We assume that $S_0^0 = s$ and $E_0^0 = \alpha s$ are constants and that

$$S_0^n = E_0^n = 0 \quad for \ n \neq 0 \tag{2.37}$$

For $n = 1$, $\Delta(1)$ is zero if ω satisfies the dispersion relation

$$-\gamma^2\omega^4 + \frac{k^2}{4}\beta^2(s_{0x})^2 + \frac{k^2}{4}\gamma\beta(1+\alpha)(s_{0y})^2 = 0 \tag{2.38}$$

Writing γ and β in terms of α, k and ω we obtain an equation cubic in ω^2

$$\begin{gathered} -(1-\tfrac{c^2k^2}{\omega^2})^2\omega^2 + [1+\alpha(1-\tfrac{c^2k^2}{\omega^2})]^2\tfrac{(s_{0x})^2}{4} + \\ (1-\tfrac{c^2k^2}{\omega^2})(1+\alpha)[1+\alpha(1-\tfrac{c^2k^2}{\omega^2})]\tfrac{(s_{0y})^2}{4} = 0 \end{gathered} \tag{2.39}$$

Eq.(2.39) is a cubic equation in ω^2, gives three roots representing three modes of propagation of the waves. One value of the phase velocity can be obtained by writing

$$(1 - \frac{c^2k^2}{\omega^2})^2\omega^2 = 0 \tag{2.40}$$

From Eq.(2.40) we obtain $\frac{\omega}{k} = c$, is a constant

This shows that the corresponding group velocity will be zero and this will represent some undamped oscillations taking place in plasma.

Expressions for the other two phase velocities can be obtained by writing

$$\begin{gathered}[1+\alpha(1-\tfrac{c^2k^2}{\omega^2})]^2\tfrac{(s_0)^2\cos^2\varphi}{4}+\\ (1-\tfrac{c^2k^2}{\omega^2})(1+\alpha)[1+\alpha(1-\tfrac{c^2k^2}{\omega^2})]\tfrac{(s_0)^2\sin^2\varphi}{4}=0\end{gathered} \tag{2.41}$$

On simplifying Eq.(2.41) we get

$$\left[1+\alpha(1-\frac{c^2k^2}{\omega^2})\right]\left[1+\alpha(1-\frac{c^2k^2}{\omega^2})\cos^2\varphi+(1+\alpha)(1-\frac{c^2k^2}{\omega^2})\sin^2\varphi\right]=0 \tag{2.42}$$

This means that

$$\left[1+\alpha(1-\frac{c^2k^2}{\omega^2})\right]=0$$

i.e.,

$$\frac{\omega}{k}=V_0=\sqrt{\frac{\alpha}{(1+\alpha)}}c \tag{2.43}$$

This equation gives one of the value of phase velocities. The other value of phase velocity can be obtained as

$$\left[1+\alpha(1-\frac{c^2k^2}{\omega^2})\cos^2\varphi+(1+\alpha)(1-\frac{c^2k^2}{\omega^2})\sin^2\varphi\right]=0 \tag{2.44}$$

On simplifying Eq.(2.44) we get

$$\frac{\omega}{k}=V_1=\sqrt{\frac{\alpha+\sin^2\alpha}{(1+\alpha)}}c \tag{2.45}$$

In the present analysis we consider the propagation mode with velocity

$$V_0=\sqrt{\frac{\alpha}{(1+\alpha)}}c$$

Now substituting Eqs.(2.12-13) and Eqs.(2.22-23) in Eqs.(2.8-2.11) and collecting and solving coefficients of different orders of ε^j for $n = 1$, we can find:

at order ε^0

The x, y and z components of Eq.(2.10) are given by

$$S_y^0 E_z^0 - S_z^0 E_y^0 = 0 \tag{2.46}$$

$$S_z^0 E_x^0 - S_x^0 E_z^0 = 0 \tag{2.47}$$

$$S_x^0 E_y^0 - S_y^0 E_x^0 = 0 \tag{2.48}$$

The x component of Eq.(2.8) is given by

$$(E_x^0 + S_x^0) = 0 \tag{2.49}$$

and the y and z components are given by

$$w^2(E_y^0 + S_y^0) = k^2 E_y^0 \tag{2.50}$$

On simplifying Eq.(2.50) we get

$$(1 - \frac{k^2}{\omega^2})E_y^0 + S_y^0 = 0 \tag{2.51}$$

Eq.(2.51) can be written as

$$(\gamma E_y^0 + S_y^0) = 0 \tag{2.52}$$

where

$$\gamma = 1 - \frac{c^2}{V^2} \tag{2.53}$$

By identifying V as $V_0 = \sqrt{\frac{\alpha}{(1+\alpha)}}c$ Eq.(2.52) can be written as

$$E_y^0 - \alpha S_y^0 = 0 \tag{2.54}$$

Similarly the z component is written as

$$E_z^0 - \alpha S_z^0 = 0 \tag{2.55}$$

At order ε^1

$$(1+\alpha)S_z^0 S_x^1 = 2V_0^2 \frac{\partial S_y^0}{\partial \xi} \tag{2.56}$$

$$(1+\alpha)S_y^0 S_x^1 = 2V_0^2 \frac{\partial S_z^0}{\partial \xi} \tag{2.57}$$

$$(E_x^1 + S_x^1) = 0 \tag{2.58}$$

$$(E_y^1 - \alpha S_y^1) = 0 \tag{2.59}$$

$$(E_z^1 - \alpha S_z^1) = 0, \tag{2.60}$$

at order ε^2

$$2V_0^2 \frac{\partial S_x^1}{\partial \xi} = [S_y^0(E_z^2 - \alpha S_z^2) - S_z^0(E_y^2 - \alpha S_y^2)] \tag{2.61}$$

$$2V_0^2 \frac{\partial S_y^1}{\partial \xi} = [S_z^0(E_x^2 - \alpha S_x^2) - S_x^0(E_z^2 - \alpha S_z^2)] \tag{2.62}$$

$$2V_0^2 \frac{\partial S_z^1}{\partial \xi} = [S_x^0(E_y^2 - \alpha S_y^2) - S_y^0(E_x^2 - \alpha S_x^2) + (1+\alpha) S_x^1 S_y^1] \qquad (2.63)$$

$$(E_x^2 + S_x^2) = 0 \qquad (2.64)$$

$$\frac{\partial(E_y^2 - \alpha S_y^2)}{\partial \xi} = -\frac{2V_0(1+\alpha)^2}{c^2}\frac{\partial S_y^0}{\partial \tau} \qquad (2.65)$$

$$\frac{\partial(E_z^2 - \alpha S_z^2)}{\partial \xi} = -\frac{2V_0(1+\alpha)^2}{c^2}\frac{\partial S_z^0}{\partial \tau} \qquad (2.66)$$

Solving for $E_y^2 - \alpha S_y^2$ and $E_z^2 - \alpha S_z^2$ from Eqs. (2.65) and (2.66) we can get

$$(E_y^2 - \alpha S_y^2) = \int_{-\infty}^{\xi} -\frac{2V_0(1+\alpha)^2}{c^2}\frac{\partial S_y^0}{\partial \tau} d\xi \qquad (2.67)$$

$$(E_z^2 - \alpha S_z^2) = \int_{-\infty}^{\xi} -\frac{2V_0(1+\alpha)^2}{c^2}\frac{\partial S_z^0}{\partial \tau} d\xi \qquad (2.68)$$

From Eq.(2.61) we know that

$$2V_0^2 \frac{\partial S_x^1}{\partial \xi} = S_y^0(E_z^2 - \alpha S_z^2) - S_z^0(E_y^2 - \alpha S_y^2) \qquad (2.69)$$

Substituting Eqs.(2.67-68) in Eq.(2.69) we get

$$2V_0^2 \frac{\partial S_x^1}{\partial \xi} = S_y^0 \left[\int_{-\infty}^{\xi} \frac{-2V_0(1+\alpha)^2}{c^2}\frac{\partial}{\partial \tau} S_z^0 d\xi\right] - S_z^0 \left[\int_{-\infty}^{\xi} \frac{-2V_0(1+\alpha)^2}{c^2}\frac{\partial}{\partial \tau} S_y^0 d\xi\right] \qquad (2.70)$$

Introducing two new variables A and θ defined by

$$S_y^0 = A\cos\theta, S_z^0 = A\sin\theta, A = S_0 \sin\ \phi, \theta \to 0 \quad as \ \ \xi \to -\infty. \qquad (2.71)$$

Substituting Eq.(2.71) in Eq.(2.56) we get

$$S_x^1 = \frac{2V_0^2}{(1+\alpha)}\frac{\partial\theta}{\partial\xi} \tag{2.72}$$

Now substituting the value of S_x^1 and using Eq.(2.71), Eq.(2.70) can be written as,

$$-\mu\frac{\partial^2\theta}{\partial\xi^2} = cos\theta\frac{\partial}{\partial\tau}\int_{-\infty}^{\xi} sin\theta d\xi - sin\theta\frac{\partial}{\partial\tau}\int_{-\infty}^{\xi} cos\theta d\xi \tag{2.73}$$

$$\mu = \frac{2V_0^3c^2}{(1+\alpha)^3}$$

Differentiating Eq.(2.73) with respect to ξ and simplifying we obtain,

$$\frac{\partial}{\partial\xi}\left[\frac{\frac{\partial\theta}{\partial\tau}+\mu\frac{\partial^3\theta}{\partial\xi^3}}{\frac{\partial\theta}{\partial\xi}}\right] = -\mu\frac{\partial^2\theta}{\partial\xi^2}\left[\frac{\partial\theta}{\partial\xi}\right] \tag{2.74}$$

Eq.(2.74) can be integrated with respect to ξ to give:

$$\frac{\frac{\partial\theta}{\partial\tau}+\mu\frac{\partial^3\theta}{\partial\xi^3}}{\frac{\partial\theta}{\partial\xi}} = -\mu\int_{-\infty}^{\xi}\frac{\partial^2\theta}{\partial\xi^2}\left[\frac{\partial\theta}{\partial\xi}\right]d\xi \tag{2.75}$$

Putting $f = \frac{\partial\theta}{\partial\xi}$, the above equation becomes

$$\frac{\partial f}{\partial\tau}+\frac{3}{2}\mu f^2\frac{\partial f}{\partial\xi}+\mu\frac{\partial^3 f}{\partial\xi^3} = 0 \tag{2.76}$$

where f is a function of ξ, and τ This equation is the modified Korteweg- deVries (mKdV) [58] equation.

2.5 Solitary wave solution

Solution of the mKdV equation can be found using the travelling wave method [59]. For applying this method, we assume the solution of the Eq. (2.76) as

$$f(\xi, \tau) = u(\zeta) \tag{2.77}$$

where $\zeta = \xi - \lambda\tau$, and λ is a constant. Substituting Eq.(2.77) in (2.76) we get

$$-\lambda u_\zeta + \frac{3}{2}\mu u^2 u_\zeta + \mu u_{\zeta\zeta\zeta} = 0 \tag{2.78}$$

Integrating Eq.(2.78) again with respect to ζ, we get

$$-\lambda u + \frac{1}{2}\mu u^3 + u_{\zeta\zeta} + A = 0 \tag{2.79}$$

Multiplying Eq.(2.79) with u_ζ and integrating again, we finally arrive at

$$u_\zeta^2 = \frac{u^4}{4} + \frac{\lambda}{\mu}u^2 - Au = 0 \tag{2.80}$$

Applying the boundary conditions $u, u_\zeta, u_{\zeta\zeta} \rightarrow 0$ as $\zeta \rightarrow \pm\infty$ which describe the solitary wave. The constant of integration can be neglected. Integrating Eq.(2.80) we have

$$\zeta - \zeta_0 = \frac{1}{2}\int \frac{du}{u\sqrt{u^2 + 4\frac{\lambda}{\mu}}} \tag{2.81}$$

which gives the solution

$$u(\zeta) = 2a^2 sech^2\,(2a\zeta) \tag{2.82}$$

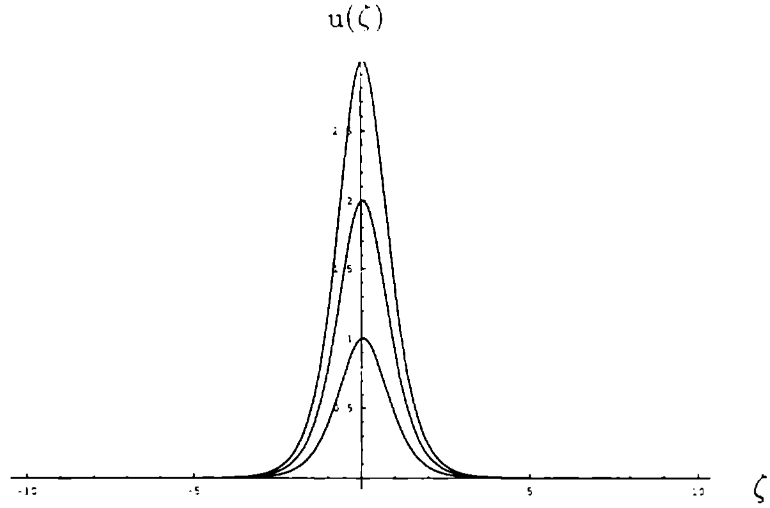

Figure 2.1: The figure shows the variation of $u(\zeta)$ for different values of a^2

With $\zeta = \xi - \lambda\tau$, $\lambda =$ constant, $a^2 = \frac{\lambda}{\mu}$ if and only if $\lambda > 0(\mu > 0)$

Fig.(2.1) shows the variation of $u(\zeta)$ with respect to ξ and τ for different values of a^2 Since $f = \frac{\partial\theta}{\partial\zeta}$, θ is defined as

$$\theta = \arccos(1 - 2sech^2 a\zeta) \tag{2.83}$$

It is seen that θ increases from 0 to 2π or decreases from 0 $to - 2\pi$ according as $a > 0$ or $a < o$ as ζ goes from $-\infty$ to ∞, since θ is given by $\theta = \int_{-\infty}^{\infty} f d\zeta$.

2.6 (2+1) dimensional case

Here all the physical quantities are assumed to depend on the space coordinates x and y and time t. We will now introduce the stretching variables ξ, ζ, and τ as

$$\xi = \varepsilon(x - Vt) \tag{2.84}$$

$$\zeta = \varepsilon^2 y \tag{2.85}$$

$$\tau = \varepsilon^3 t \tag{2.86}$$

By giving the same boundary conditions and the perturbation expansion of the dependent variables, we will arrive at the same set of equations as in the case of (1+1) dimensions for orders $\varepsilon^0, \varepsilon^1$, but we have at order ε^2

$$2V_0^2 \frac{\partial S_x^1}{\partial \xi} = [S_y^0(E_z^2 - \alpha S_z^2) - S_z^0(E_y^2 - \alpha S_y^2)] \tag{2.87}$$

$$2V_0^2 \frac{\partial S_y^1}{\partial \xi} = [S_z^0(E_x^2 - \alpha S_x^2) - S_x^0(E_z^2 - \alpha S_z^2)] \tag{2.88}$$

$$2V_0^2 \frac{\partial S_z^1}{\partial \xi} = [S_x^0(E_y^2 - \alpha S_y^2) - S_y^0(E_x^2 - \alpha S_x^2) + (1+\alpha) S_x^1 S_y^1] \tag{2.89}$$

$$(E_x^2 + S_x^2) = 0 \tag{2.90}$$

$$\frac{\partial(E_y^2 - \alpha S_y^2)}{\partial \xi} = -\frac{2V_0(1+\alpha)^2}{c^2} \frac{\partial S_y^0}{\partial \tau} - (1+\alpha) \frac{\partial^2 E_y^0}{\partial \zeta^2} \tag{2.91}$$

$$\frac{\partial(E_z^2 - \alpha S_z^2)}{\partial \xi} = -\frac{2V_0(1+\alpha)^2}{c^2} \frac{\partial S_z^0}{\partial \tau} - (1+\alpha) \frac{\partial^2 E_z^0}{\partial \zeta^2} \tag{2.92}$$

Solving for $E_y^2 - \alpha S_y^2$ and $E_z^2 - \alpha S_z^2$, from Eqs.(2.91) and (2.92) we can get

$$(E_y^2 - \alpha S_y^2) = \int_{-\infty}^{\xi} -\frac{2V_0(1+\alpha)^2}{c^2} \frac{\partial S_y^0}{\partial \tau} d\xi + \int_{-\infty}^{\xi} \int_{-\infty}^{\xi} \alpha(1+\alpha) \frac{\partial^2 S_y^0}{\partial \zeta^2} (d\xi)^2 \tag{2.93}$$

$$(E_z^2 - \alpha S_z^2) = \int_{-\infty}^{\xi} -\frac{2V_0(1+\alpha)^2}{c^2}\frac{\partial S_z^0}{\partial \tau} d\xi - \int_{-\infty}^{\xi}\int_{-\infty}^{\xi} \alpha(1+\alpha)\frac{\partial^2 S_z^0}{\partial \zeta^2}(d\xi)^2 \quad (2.94)$$

From Eq.(2.87) we know that

$$2V_0^2\frac{\partial S_x^1}{\partial \xi} = S_y^0(E_z^2 - \alpha S_z^2) - S_z^0(E_y^2 - \alpha S_y^2) \quad (2.95)$$

Substituting for

$$(E_y^2 - \alpha S_y^2) \;\; and \;\; (E_z^2 - \alpha S_z^2) \quad (2.96)$$

in Eq.(2.95) we get

$$\begin{aligned} 2V_0^2\tfrac{\partial S_x^1}{\partial \xi} = S_y^0 &\left[\textstyle\int_{-\infty}^{\xi} -\frac{2V(1+\alpha)^2}{c^2}\frac{\partial S_z^0}{\partial \tau}d\xi - \int_{-\infty}^{\xi}\int_{-\infty}^{\xi}\alpha(1+\alpha)\frac{\partial^2 S_z^0}{\partial \zeta^2}(d\xi)^2\right] \\ -S_z^0 &\left[\textstyle\int_{-\infty}^{\xi} -\frac{\partial S_y^0}{\partial \tau}d\xi + \int_{-\infty}^{\xi}\int_{-\infty}^{\xi}\alpha(1+\alpha)\frac{\partial^2 S_y^0}{\partial \zeta^2}(d\xi)^2\right] \end{aligned} \quad (2.97)$$

Introducing two new variables A and θ defined by

$$S_y^0 = A\cos\theta, S_z^0 = A\sin\theta, A = S_0\sin\phi, \theta \to 0 \;\; as \;\; \xi \to -\infty. \quad (2.98)$$

Substituting Eq.(2.98) in (2.56) we have

$$S_x^1 = \frac{2V_0^2}{(1+\alpha)}\frac{\partial \theta}{\partial \xi} \quad (2.99)$$

Now substituting the value of S_x^1 and using Eq.(2.98), Eq.(2.97) can be written as,

$$\begin{aligned} -\mu\frac{\partial^2\theta}{\partial \xi^2} = cos\theta\frac{\partial}{\partial \tau}\int_{-\infty}^{\xi} sin\theta d\xi + \sigma cos\theta\frac{\partial^2}{\partial \zeta^2}\int_{-\infty}^{\xi}\int_{-\infty}^{\xi} sin\theta(d\xi)^2 \\ -sin\theta\frac{\partial}{\partial \tau}\int_{-\infty}^{\xi} cos\theta d\xi - \sigma sin\theta\frac{\partial^2}{\partial \zeta^2}\int_{-\infty}^{\xi}\int_{-\infty}^{\xi} cos\theta(d\xi)^2 \end{aligned} \quad (2.100)$$

$$\mu = \frac{2V_0^3 c^2}{(1+\alpha)^3}$$

$$\sigma = \frac{\alpha c^2}{2V_0(1+\alpha)}$$

Differentiating (2.100) with respect to ξ and simplifying we obtain ,

$$\frac{\partial \theta}{\partial \tau} + \mu \frac{\partial^3 \theta}{\partial \xi^3} + \sigma \left[cos\theta \frac{\partial^2}{\partial \zeta^2} \int sin\theta d\xi - sin\theta \frac{\partial^2}{\partial \zeta^2} \int_{\infty}^{\xi} cos\theta d\xi \right] = -\mu \frac{\partial^2 \theta}{\partial \xi^2} [\frac{\partial \theta}{\partial \xi}]^2 + \sigma \left[cos\theta \frac{\partial^2}{\partial \zeta^2} \int \cos\theta d\xi + sin\theta \frac{\partial^2}{\partial \zeta^2} \int sin\theta d\xi \right] \frac{\partial \theta}{\partial \xi} \quad (2.101)$$

On simplifying Eq.(2.101) we obtain:

$$\frac{\partial}{\partial \xi} \left[\frac{\frac{\partial \theta}{\partial \tau} + \mu \frac{\partial^3 \theta}{\partial \xi^3} - \sigma \int \frac{\partial^2 \theta}{\partial \zeta^2}}{\frac{\partial \theta}{\partial \xi}} \right] = -\mu \frac{\partial^2 \theta}{\partial \xi^2} [\frac{\partial \theta}{\partial \xi}] \quad (2.102)$$

Eq.(2.102) can be integrated with respect to ξ to give:

$$\frac{\frac{\partial \theta}{\partial \tau} + \mu \frac{\partial^3 \theta}{\partial \xi^3} - \sigma \int_{-\infty}^{\xi} \frac{\partial^2 \theta}{\partial \zeta^2}}{\frac{\partial \theta}{\partial \xi}} = -\mu \int_{-\infty}^{\xi} \frac{\partial^2 \theta}{\partial \xi^2} \left[\frac{\partial \theta}{\partial \xi} \right] d\xi \quad (2.103)$$

Putting $f = \frac{\partial \theta}{\partial \eta}$, the above equation becomes

$$\frac{\partial f}{\partial \tau} + \frac{3}{2} \mu f^2 \frac{\partial f}{\partial \xi} + \mu \frac{\partial^3 f}{\partial \xi^3} = \sigma \int_{-\infty}^{\xi} \frac{\partial^2 f}{\partial \zeta^2} \quad (2.104)$$

where f is a function of ξ, ζ and τ.

Differentiating Eq.(2.104) with respect to ξ we get,

$$\left[\frac{\partial f}{\partial \tau} + \frac{3}{2} \mu f^2 \frac{\partial f}{\partial \xi} + \mu \frac{\partial^3 f}{\partial \xi^3} \right]_{\xi} = \sigma \frac{\partial^2 f}{\partial \zeta^2} \quad (2.105)$$

This equation is the modified Kadomtsev- Petviashvili (mKP) equation [66]. Solution of mKP equation can be found using the travelling wave method. For applying this, we assume the solution of the equation (2.105) as

$$f(\xi, \zeta, \tau) = u(\eta) \tag{2.106}$$

where $\eta = \xi + \zeta - \lambda\tau$, and λ is a constant. Substituting Eq.(2.106) in (2.105) we get

$$\frac{\partial}{\partial\eta}\left(-\lambda u_\eta + \frac{3}{2}\mu u^2 u_\eta + \mu u_{\eta\eta\eta}\right) - \sigma u_{\eta\eta} = 0 \tag{2.107}$$

On integrating Eq.(2.107), we obtain

$$-\lambda u_\eta + \frac{3}{2}\mu u^2 u_\eta + \mu u_{\eta\eta\eta} - \sigma u_\eta + A = 0 \tag{2.108}$$

where A is the constant of integration. Integrating Eq.(2.108) with respect to η, we get

$$-\lambda u + \frac{1}{2}\mu u^3 + u_{\eta\eta} - \sigma u + Au + B = 0 \tag{2.109}$$

Multiplying Eq.(2.109) with u_η and integrating again, we finally arrive at

$$u_\eta^2 = u^4/4 + \frac{(\lambda - \sigma - A)}{\mu}u^2 - 2Bu - 2C = 0 \tag{2.110}$$

where B and C are two integration constants. At this stage let us impose the boundary conditions $u, u_\eta, u_{\eta\eta} \to 0$ as $\eta \to \pm\infty$, which describe the solitary wave .Thus the constants of integration can be neglected. On integrating Eq.(2.110) we obtain,

$$\eta - \eta_0 = \frac{1}{2}\int \frac{du}{u\sqrt{u^2 + 4(\frac{\lambda-\sigma}{\mu})}} \tag{2.111}$$

which gives the solution

$$u(\eta) = -2\frac{(\lambda - \sigma)}{\mu} sech^2\left[\sqrt{\frac{(\lambda - \sigma)}{\mu}}\right](\eta - \eta_0) \tag{2.112}$$

where η_0 is an arbitrary constant of integration, it merely denotes the position of the peak at t=0. It plays a minor role. So that we can write

$$u(\eta) = -2k^2 sech^2 2k\eta \tag{2.113}$$

where $k^2 = \sqrt{\frac{(\lambda-\sigma)}{\mu}}$.

Since $f = \frac{\partial\theta}{\partial\eta}$, θ is obtained as $\theta = \int_{-\infty}^{\eta} f d\eta$. θ increases from 0 to 2π or decreases from 0 to -2π according as $k > 0$ or $k < 0$ as η goes from $-\infty$ to $+\infty$. We can obtain explicit form of solutions for S_y^0, S_z^0, E_y^0, E_z^0, E_x^1 and S_x^1 from Eqs.(2.71-2.72). Thus we can write

$$S_y^0 = S_0 sin\phi \cos(2k^2\sqrt{[1 - sech^2 k\eta]}) \tag{2.114}$$

$$S_z^0 = S_0 sin\phi \sin(2k^2\sqrt{[1 - sech^2 k\eta]}) \tag{2.115}$$

$$E_y^0 = \alpha S_0 \sin\phi \cos(2k^2\sqrt{[1 - sech^2 k\eta)]}) \tag{2.116}$$

$$E_z^0 = \alpha S_0 \sin\phi \sin(2k^2\sqrt{[1 - (sech^2 k\eta)]}) \tag{2.117}$$

$$E_x^1 = \frac{2Vk^2 sech^2 k\eta}{(1+\alpha)} \tag{2.118}$$

$$S_x^1 = -\frac{2Vk^2 sech^2 k\eta}{(1+\alpha)} \tag{2.119}$$

We have plotted the components of displacement vector field in Figs.(2.2)-(2.4).

2.7 Conclusion

We have considered the propagation of an electromagnetic waves in a cold collision free plasma and studied the dynamics of the system when the electromagnetic wave passes

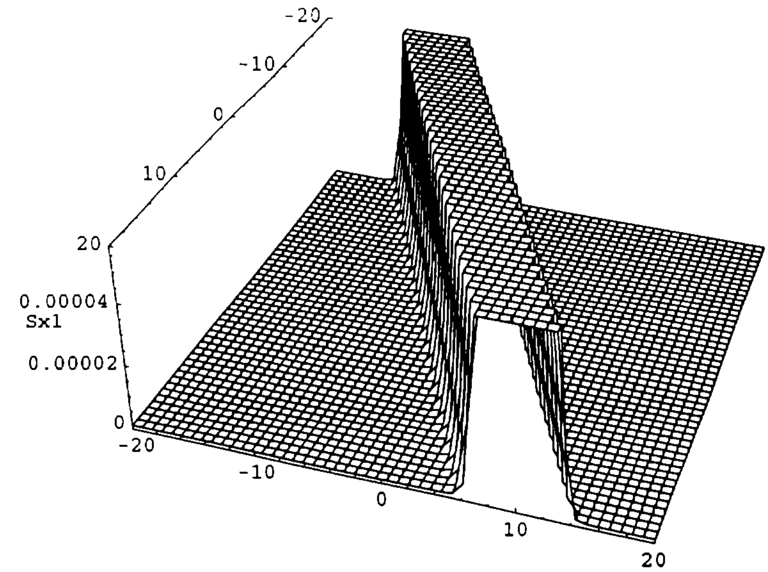

Figure 2.2: The figure shows the variation of S_x^1 vs η.

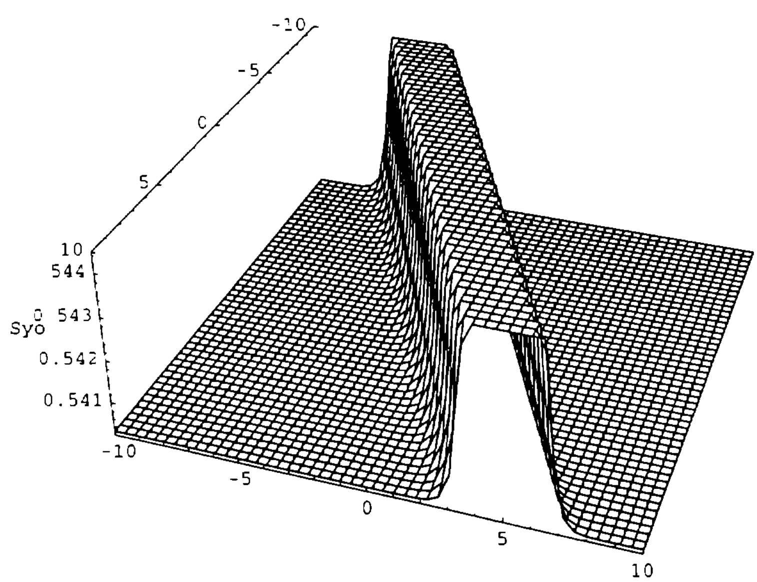

Figure 2.3: The figure shows the variation of S_y^0 vs η.

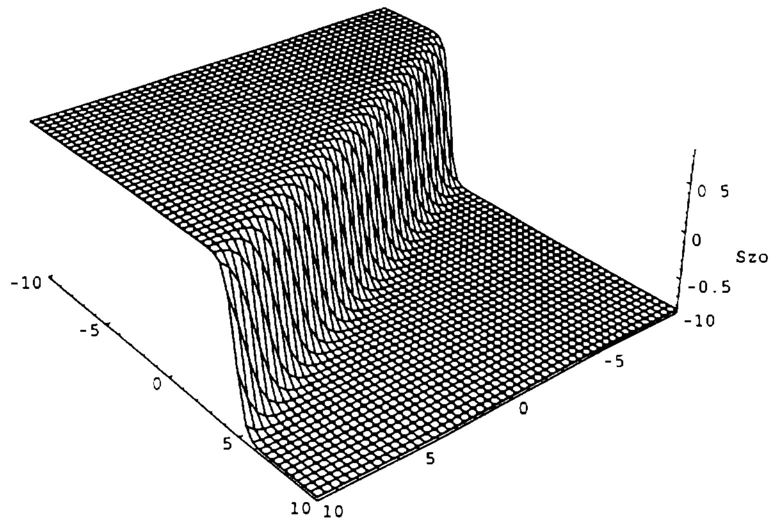

Figure 2.4: The figure shows the variation of S_z^0 vs η.

through the medium.

In the case of plasma it is important to formulate a systematic perturbational approach, in which the lowest order terms account for the nonlinear self interaction, while the successive higher order determine the nonlinear mode- mode coupling effects. This type of interactions is essential for the formation of soliton like structures in plasma and for explaining strong plasma turbulence. In the higher order contributions, the nonlinear effects describe interaction among the first order terms. In the frame work of the reductive perturbation theory, the higher order nonlinear effects describe the interaction among the lowest order nonlinear terms at the same time, the higher order dispersive terms competes with the nonlinear effect among the lowest order nonlinear terms. So we can conclude that it is the lowest order nonlinear terms account for the formation of solitons while the higher order nonlinear terms results the distortion of the soliton structure as a result of the soliton soliton interactions. The solitary wave interaction in plasma is studied in the fourth chapter of the thesis.

We have studied the nature of fluctuations arises due to the interaction of the electric field associated with the electromagnetic wave and the displacement vector field. In (1+1) dimensions it is found that the nonlinear evolution equations give soliton solutions and is integrable via Paileve analysis [section 1.4]. But when we go to higher dimensions that is in (2+1) dimensions the nonlinear evolution equations is found to be non-integrable in the sense of Painleve analysis but we can find solitary wave solutions. We found that the S_x^1, E_x^1, S_y^0 and E_y^0 components give soliton solutions while the S_z^0 and E_z^0 components give kinks in (2+1) dimensional case. We can see that the displacement vector field and the electric field are spatially localized.

Chapter 3

Stability analysis of modified KP equation

There is something fascinating about science.
One gets such wholesale returns of conjecture out
of such a trifling investment of fact.

Mark Twain

3.1 Introduction

In chapter 2 the propagation of electromagnetic waves through a cold collisionless plasma in (1+1) and in (2+1) dimensions has been studied. Starting from the basic equations of plasma, treating plasma as an electron fluid, using the reductive perturbation method we have reduced the system of equations to modified Koteweg-deVries (mKdV) equation in (1+1) diemnsions and modified Kadomtsev-Petviashvili (mKP) [66] equation in (2+1) diemnsions. We have found the solitary travelling wave solutions for the mKP The nonlinearity inherent in most classical equations of motion makes the question of stability and the prediction of long-term behaviour of the system all the more interesting.

The concept of stability arises from a formal consideration of the wave function. In linear wave theory the amplitude of the waves is much less than the stationary state vector, so that the wave can be considered as a small disturbance. We consider th disturbance δn of the density n, such that $\delta n(x,t) << n_0$, where n_0 is a function of space and time, but it is assumed that its variation is much slower than that of the disturbance. If this is the case, the wave function can be represented by a super position of plane waves oscillating at frequency $\omega(k)$, where ω is the solution of the linear dispersion $D(\omega, k) = 0$. Any wave field component $\delta A(x,t)$ can then be Fourier decomposed as [65]

$$\delta A(x,t) = \sum_k A_k exp(ik.x - i\omega t). \tag{3.1}$$

The first step towards the stability analysis of a wave is the linearization of the equation about some equilibrium point and the condition that the linearized equations have non-trivial solutions leading to a dispersion relation. For an infinite media the wave number must be real. Then the nature of the perturbation depends on ω. In general the dispersion relation is a complex equation and has a number of complex frequency solutions: $\omega = \omega_r + i\gamma$. From Eq.(3.1) it is clear that, for real ω, the disturbances are oscillating waves. On the other hand, for complex solutions the behaviour of the wave amplitude depends heavily on the sign of the imaginary part γ of the frequency. If $\gamma < 0$ the real part of the amplitude becomes an exponentially decreasing function of time, and the wave is damped. On the other hand, for $\gamma > 0$ the wave amplitude grows exponentially in time, and we encounter a linear instability. In this case the decrement γ is called the *growth rate* of the corresponding eigen mode. That is, instability can only arise if there are free energy sources in the plasma which feed the growing waves. If this is not the case, then a solution with a positive γ is a fake solution which violates energy conservation and causality. In general the solution of a dispersion relation will be of the form $\omega = \omega_r + i\gamma$ with ω_r and γ both non-zero real quantities. Then for $\gamma > 0$ we have the case of a wave propagating with phase velocity $\frac{\omega_r}{k}$ and with an amplitude increasing exponentially with time ($e^{\gamma t}$). For $\gamma < 0$ the wave eventually damps away. For $\gamma \succ 0$, due to the exponential increase in amplitude, the disturbance will be comparable to the equilibrium values. When this happens the entire procedure is bound to break down. Thus the study of instabilities using linear theory is justifiable for short times ($\gamma t < 1$), the larger the value of γ the shorter is the time.

The amplitude of an unstable wave increases as

$$A_k(t) = A_k \exp[\gamma(\omega_r, k)t] \tag{3.2}$$

Thus the linear approximation breaks down when the amplitude becomes comparable to the background value of the field, i.e., $A_k(t)/A_0 \approx 1$ or at the nonlinear time

$$t_{nl} \approx \gamma^{-1} \ln(\frac{A_0}{A_k}) \tag{3.3}$$

The linear approximation for unstable modes holds only for times $t << t_{nl}$.When the linear approximation is violated, other processes set on which are called nonlinear because they involve interaction of the waves with each other and within the background plasma, which cannot be treated by linear methods. The time t_{nl} reached is the earlier the larger the growth rate is. When the growth rate becomes larger than the frequency, $\gamma > \omega$, the wave amplitude explodes and the wave has no time to perform even one single oscillation during one wave period. The wave concept becomes obsolete and in this case and it is reasonable to consider in the first place only the instabilities of comparably smaller growth rate which satisfies the condition of linearity during many wave periods

$$\frac{\gamma}{\omega} << 1$$

Recently quite a lot of progress has been made in the field of nonlinear waves, however the problem of stability is still open. There are a number of methods for studying the stability problem of nonlinear waves. In Whitham's [61] original method the dynamics of the problem can be described in conservation form. Another method for studying the stability problem is Haye's Hamiltonian method [75]

One study that does resolve the stability problem of nonlinear waves with arbitrary amplitude is provided by the elegant work of Rowlands [69]. In this procedure, conservation laws, Lagrangian and Hamiltonians are not required. Several authors have carried out the stability analysis of nonlinear waves using this method [69-81]. The instability of the wave can be due to various reasons such as the system being driven or pumped with energy through some mechanisms like a rotation, a background flow or a heat gradient There may be a 'control parameter' within the mathematical model, whose role can be important in that the system in question may become unstable under the influence of the background energy flow when this parameter passes through a critical value. The infinitesimally small disturbances will grow if the control parameter is in the post- critical region.

The method initiated by Rowlands and later developed by Infeld and Rowlands, now called the K- perturbation expansion method and can be used for for finding the stability of solitary wave solutions of a certain class of nonlinear equations based on the assumption that the original nonlinear structure undergoes a long wavelength perturbation. In this chapter we have studied the stability of the solutions of the modified Kadomtsev-Petviashvili equation (mKP) obtained in chapter 2 [66] by applying a periodic perturbation at an angle θ to the direction of propagation of the steady state solution. The key feature of this method is the regrouping of algebraically secular terms in order to obtain a spatially bounded solution with the correct asymptotic behaviour.

3.2 K-expansion

In this section, the method of K-expansion is described using nonlinear Klein-Gordon equation as a model equation. In this procedure we first transform the equation into a form in which the basic nonlinear structure is a function of one variable only [69].

Let us now consider the nonlinear Klein-Gordon equation.

$$(\partial_T^2 - \partial_X^2)\phi + V'(\phi) = 0 \tag{3.4}$$

Thus, if ϕ is the basic nonlinear structure, depends on $X = x - Ut$, we take

$$X = \frac{x - Ut}{\sqrt{U^2 - 1}}; \quad T = \frac{t - Ux}{\sqrt{U^2 - 1}} \tag{3.5}$$

We now perturb $\phi(X)$ in Eq.(3.4) and linearize in $\delta\phi$. The perturbation is of the form

$$\delta\phi \sim \tilde{\delta\phi} e^{i(KX - \Omega T)} \tag{3.6}$$

So we can write Eq.(3.4) as (by reverting to lower case x)

$$(\partial_T^2 - \partial_x^2)\delta\phi - V''\delta\phi = 0 \tag{3.7}$$

and substituting Eq.(3.6) in Eq.(3.1) we find:

$$(\Omega^2 + \partial_x^2)\delta\phi + V''\delta\phi = 0 \tag{3.8}$$

$\delta\phi$ in general will be a periodic function of x with the same period λ as $\phi(X, A)$, multiplied by e^{iKx}, K being real real. Thus $\delta\tilde{\phi}$ in Eq.(3.6) is a λ periodic function. K

and $K + 2\pi n\lambda$ will give the same Ω. We now assume both K and Ω to be small and of the same order and expanding all quantities in K

$$\Omega = K\Omega_1 + K^2\Omega_2 +$$

$$\delta\tilde{\phi} = \delta\phi_0 + K\delta\phi_1 + K^2\delta\phi_2 + \tag{3.9}$$

At order K^0 we have

$$L\delta\phi_0 = (\partial_x^2 + V'')\delta\phi_0 = 0 \tag{3.10}$$

The solution of Eq.(3.10) can be found in the following way [69]:

Integrating Eq.(3.4) we get

$$\frac{1}{2}\phi_x^2 + V(\phi) = A \tag{3.11}$$

Differentiating Eq.(3.11) with respect to x, dividing through out by ϕ_x, and differentiating once again, we get

$$(\partial_x^2 + V'')\frac{\partial\phi}{\partial x} \tag{3.12}$$

From Eqs.(3.10) and (3.12) we can write

$$\delta\phi_0 = \frac{\partial\phi}{\partial x} \tag{3.13}$$

At order K^1 :we have the equation

$$L\delta\phi_1 = (\partial_x^2 + V''\delta\phi_1) = -2i\phi_{xx} \tag{3.14}$$

The solution of Eq.(3.14) is given by

$$\delta\phi_1 = -ix\phi_x + \beta\phi_A \tag{3.15}$$

At order K^2 the equation is given by

$$L\delta\phi_2 + (\Omega_1^2 - 1)\phi_x - 2i\frac{\partial}{\partial x}(ix\phi_x + i\lambda\lambda_A^{-1}\phi_A) = 0 \tag{3.16}$$

Upon multiplication by ϕ_x and integration over a wavelength, we get

$$\int \phi_x L\delta\phi_2 dx + (\Omega_1^2 - 1)\int \phi_x^2 dx - 2i\int \phi_x\partial_x(ix\phi_x + i\lambda\lambda_A^{-1}\phi_A)dx = 0 \tag{3.17}$$

Using the fact that L is self-adjoint and ϕ_x is its eigenfunction, we readily obtain

$$\Omega_1^2\int \phi_x^2 dx + 2\lambda\lambda_A^{-1}\int \phi_x\phi_{Ax}dx = 0 \tag{3.18}$$

In spite of L being second order, there is no other consistency condition. Now introduce

$$g = \frac{G}{\sqrt{U^2 - 1}} = \int \phi_x^2 dx = 2\int_{\phi_1}^{\phi_2} \phi_x d\phi \tag{3.19}$$

$\phi_x = 0$ at ϕ_1 and ϕ_2. It is easily seen that $g_A = 2\lambda$. Thus, we have

$$\Omega_1^2 = -g_A/gg_{AA} = -G'^2/GG'' \tag{3.20}$$

$$\Omega_1 = \lim_{K\to 0}.\frac{\Omega}{K} = \frac{\partial\Omega}{\partial K} = \pm[(G')^2/GG'']^{\frac{1}{2}} = V_{CH}(0) \tag{3.21}$$

The characteristic velocity is $\pm[(G')^2/GG'']^{\frac{1}{2}}$ when this quantity is real. If it is imaginary, the perturbation grows as $e^{|\Omega_1|t}$ and the basic nonlinear wave $\phi(x, A, U)$will be unstable. The stability of shock and soliton solutions can be treated by performing the calculations for nonlinear waves and then taking A$\rightarrow$ $A_{soliton}$or A$\rightarrow$ A_{Shock} as $A \rightarrow \infty$.

3.3 Stability analysis of mKP equation

To study the stability of the solution of the mKP equation, we apply a periodic perturbation at an angle θ to the direction of propagation of the steady state solution. If K and Ω are the wave vector and frequency of the perturbation then it is assumed that $\frac{\Omega}{\omega} \approx K/k$ and K<<k where K and Ω are the wave vector and frequency of the perturbation while k and ω describe the basic nonlinear wave structure. In order to carry out this method the first step is to linearize the equation around a stationary nonlinear wave. For this we first transform the equation into a form in which the basic nonlinear structure is a function of one variable only. The form of modified Kadomtsev-Petviashvili equation is given by Eq.(1.107) as

$$\left[\frac{\partial U}{\partial \tau} + \gamma U^2 \frac{\partial U}{\partial \xi} + \beta \frac{\partial^3 U}{\partial \xi^3}\right]_{\xi} = \sigma \frac{\partial^2 U}{\partial \zeta^2} \tag{3.22}$$

We now seek the steady state solution of this mKP equation in the form $U = U_0(X)$ and also applying a transformation for the independent variable as

$$X = \xi - c\tau \tag{3.23}$$

Using the transformation given by Eq.(3.23), we can write the mKP equation in steady state form as

$$-c\frac{dU_0}{dX} + \gamma U_0^2 \frac{dU_0}{dX} + \beta \frac{d^3 U_0}{dX^3} = 0 \tag{3.24}$$

The steady state solution of this equation has been found to be

$$U_0(X) = U_m sech \mu X \tag{3.25}$$

where

$$U_m = \frac{3c}{\gamma}, \qquad \mu = \sqrt{\frac{c}{\beta}}$$

The effect of perturbation of the solution can be studied by writing

$$U(\xi,\zeta,\tau) = U_0(X) + u(X,\zeta,\tau), \tag{3.26}$$

in Eq.(3.22) where $U_0(X)$ is given by (3.25) and u for a long wavelength plane wave perturbation in a direction with direction cosines (l_1, l_2), is given by

$$u = u(X)\exp i[K(l_1\xi + l_2\zeta) - \Omega\tau] \tag{3.27}$$

where u(X) is a complex periodic function of ξ with the same period λ as $U_o(X)$,(l_1, l_2) $=l(cos\theta, sin\theta)$, l is a real constant, θ is the angle at which the perturbation is applied with the direction of propagation, Ω is a constant (possibly complex). Now we introduce expansions for u(X) and Ω in terms of K as

$$u(X) = u_0(X) + Ku_1(X) + K^2u_2(X) + \tag{3.28}$$

$$\Omega = K\Omega_1 + K^2\Omega_2 + \tag{3.29}$$

Substituting Eq.(3.27) and using the expansions given by (3.28) and (3.29), Eq. (3.22) can be linearized with respect to u yields:

$$\begin{aligned} \frac{\partial}{\partial X}[\beta\frac{\partial^2}{\partial X^2} + \gamma U_m^2 - c]u_0(X) = \\ i[\omega + cKl_1 - \gamma Kl_1U_m^2 + K^3\beta l_1^3]u_0(X) + \\ 3K^2\beta l_1^2 u_0(X)_X - 3iK\beta l_1 u_0(X)_{XX} + \sigma\int Kl_2^2 u_0(X)dX \end{aligned} \tag{3.30}$$

Our aim is to find Ω_1 and this can be done by solving the resulting equations formed by zeroth, first and second order coefficients of K. The equation formed by the zeroth order coefficients of K is:

$$[-c+\gamma U_m^2]u_0+\beta\frac{\partial^2 u_0}{\partial X^2} = A \qquad (3.31)$$

where A is an integration constant. It is clear from the above equation that the homogeneous part of this equation has two linearly independent solutions.

$$f=\frac{\partial U_0}{\partial X} \qquad (3.32)$$

and

$$g=f\int^X \frac{dX}{f^2} \qquad (3.33)$$

The general solution of the zeroth order equation can be written as

$$u_0 = A_1 f + A_2 g - Af\int^X \frac{g}{W}dX + Ag\int^X \frac{f}{W}dX \qquad (3.34)$$

where A_1 and A_2 are two constants and W is the Wronskian. The general solution of the zeroth order equation must remain finite as $X \to \pm\infty$ which requires that A_2=A=0. Hence the solution u_0 becomes

$$u_0 = A_1 f \qquad (3.35)$$

The first order equation, i.e, the equation with terms linear in K is given by

$$[-c+\gamma U_m^2]u_1+\beta\frac{\partial^2 u_1}{\partial X^2} = iA_1(a_1+b_1 tan^2\mu X)U_0^2 + B \qquad (3.36)$$

$$a_1 = (\Omega_1 + cl_1) - \frac{1}{2}U_m^2\mu_1 + 2\mu^2\mu_2 \qquad (3.37)$$

$$b_1 = \frac{1}{2}U_m^2\mu_1 - 6\mu^2\mu_2 \tag{3.38}$$

where

$$\mu_1 = \gamma l_1, \quad \mu_2 = 3\beta l_1$$

Now following the same procedure used for evaluating u_0, the general solution of the first order equation, for u_1 not tending to $\pm\infty$ as $X \to \pm\infty$, can be written as

$$u_1 = B_1 f - iA_1\mu[(a_1 + b_1)Xf + \frac{1}{3}(3a_1 + b_1)U_0^2] \tag{3.39}$$

where B is an integration constant. The equation with terms involving K^2 is given by,

$$\begin{aligned}[-c\frac{\partial}{\partial X} + \gamma U_m^2\frac{\partial}{\partial X} + \beta\frac{\partial^3}{\partial X^3}]u_2 &= i\omega_2 u_0 \\ &+i(\omega_1 + cl_1)u_1(X) - \mu_1 u_1(X) - \\ &\mu_2\frac{\partial^2 u_1}{\partial X^2} + \mu_3\frac{\partial}{\partial X}u_0 + \mu_4\int u_0 dX\end{aligned} \tag{3.40}$$

where

$$\mu_3 = 3\beta l_1^2, \qquad \mu_4 = \sigma l_2^2$$

A solution of this second order equation exists if the right hand side of (3.40) is orthogonal to a kernel of the adjoint operator

$$-c\frac{\partial}{\partial X} + \gamma U_m^2\frac{\partial}{\partial X} + \beta\frac{\partial^3}{\partial X^3} \tag{3.41}$$

This kernel must tend to zero as $X \to \pm\infty$. Thus we can write the following equation determining ω_1:

$$\int^X [U_0 i\omega_2 u_0 + i(\Omega_1 + cl_1)u_1(X) - \mu_1 u_1(X) - \mu_2 \frac{\partial^2 u_1}{\partial X^2} + \mu_3 \frac{\partial u_0}{\partial X} + \mu_4 \int u_0 dX]dX = 0 \quad (3.42)$$

Now substituting the expressions for u_0 and u_1 given by (3.35) and (3.39) in (3.42) and then performing the integration we arrive at the following dispersion relation:

$$\Omega_1 = D - l_1 c + (D^2 - R)^{\frac{1}{2}} \quad (3.43)$$

where

$$D = 2/3[U_m \mu_1 - 2\mu_2 \mu^2] \quad (3.44)$$

$$R = 16/45[3U_m^2 \mu_1^2 - 3U_m \mu_1 \mu_2 \mu^2 - 3\mu_2^2 \mu^4 + \mu_3 \mu^4 + 4\mu_4] \quad (3.45)$$

From Eq.(3.43) we can see that the solutions are stable if $R - D^2 < 0$. The growth rate of instability is defined as

$$\Gamma = (R - D^2)^{\frac{1}{2}} = l_1^2[\Omega_1^2 - \frac{5}{3}(1 + \Omega_1^2)tan^2\delta] \quad (3.46)$$

where δ is the angle at which the perturbation is applied with the direction of propagation.

We have plotted the graphs showing the Ω_1 ranges for different values of δ for which the solitary waves become stable and unstable and is shown in Figs. 3.1-3.3.

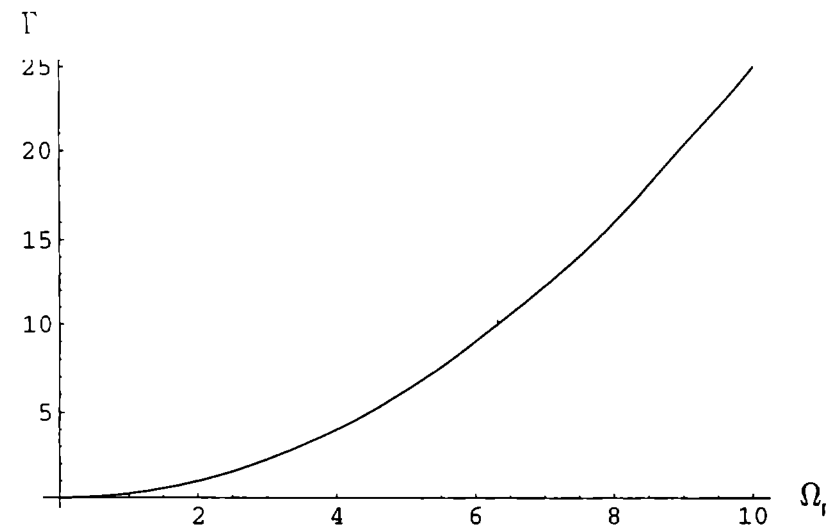

Figure 3.1: The figure shows Ω_1 ranges for $\delta = 0$.

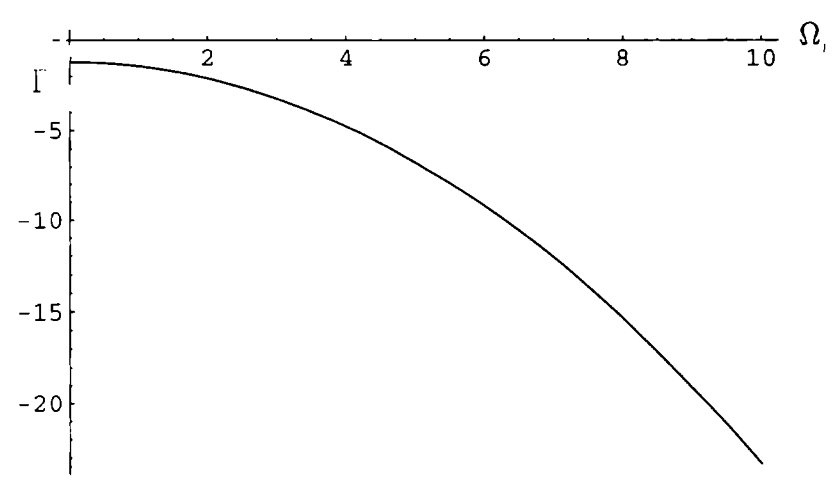

Figure 3.2: The figure shows Ω_1 ranges for $\delta = 15$.

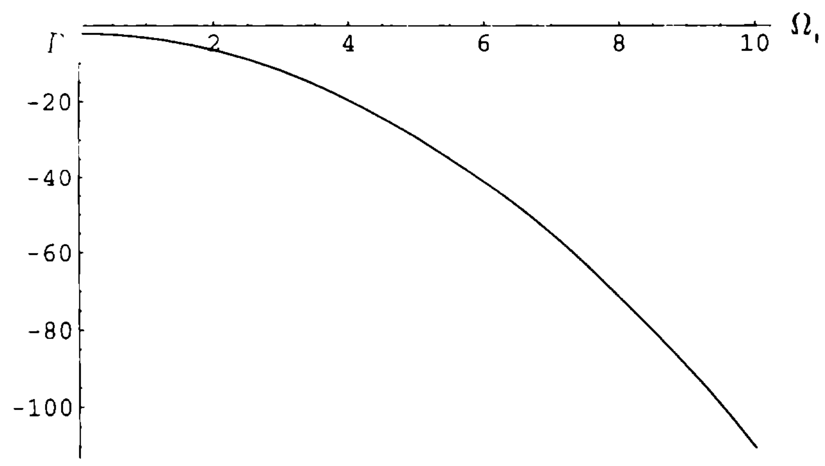

Figure 3.3: The figure shows Ω_1 ranges for $\delta = 40$.

3.4 Conclusion

Stability analysis of solitary waves has been studied using the K- perturbation expansion that is valid for small but finite amplitude solitary waves and long wave length perturbation modes. There is instability if the propagation angle the solitary wave makes with the external magnetic field is sufficiently small, there is still instability if the direction of propagation of the perturbation falls in a certain region. This region is bounded by two planes passing through the direction of the external magnetic field and does not contain the direction in which the solitary wave propagation. The results of our present investigation may be useful in understanding the nonlinear features of localized electromagnetic disturbances in many laboratory and space plasma situations. Since in many astrophysical situations there are extremely large amplitude solitary waves and short wave length perturbation modes, we propose to develop a more exact theory for stability analysis of arbitrary amplitude solitary waves and arbitrary wavelength perturbation modes through a generalization of the present work to such waves and modes.

Chapter 4

Interaction of solitary waves in a cold collisionless plasma

The opposite of a correct statement is a false statement. But the opposite of a profound truth may well be another profound truth.

N. Bohr

4.1 Introduction

Several authors have studied nonlinear wave propagation through plasma . The starting point is the famous Vlasov-Poisson self-consistent field problem The well known integrable evolution equations like Kortweg-deVries equation and nonlinear Schrodinger equation can be derived from the Vlasov-Poisson field equations using different approximations [65].

Interaction between self- focusing cylindrical beams (spatial solitons) in bulk media is a problem of obvious interest both by itself and for applications. The interaction between optical solitons influences directly the capacity and quality of communication process [82].The role of soliton resonance in the onset of plasma turbulence and the possibility of the nonlinear explosion mode are of current interest to both theoretical physicists and experimentalists [83-84]. In the study of the interactions between two obliquely moving solitons, described by the shallow water wave equation, Miles [85] has shown that at certain angles of intersection, the two obliquely directed solitons interact strongly to make a branch soliton from a point at which the wave fronts of the two solitons meet together. This is called the resonant interaction of solitons. Newell and Redkopp [87] proposed the interaction on the framework of the Zakharov- Shabat [86] theory of integrable systems with higher spatial dimensions.

In systems described by equations of the same family like Korteweg-deVries equations, characterized by dispersion effects only, solitons constitute a natural fundamental eigenmode system of waves. When a wave evolves into a chain of solitons of different amplitudes, widths and speeds, the plasma assumes a grainy quasi-irregular structure, which cannot be described anymore by weak turbulence theory. This is the origin of the notion that the soliton state of a plasma corresponds to the state of strong plasma turbulence. In this state the plasma is filled with localized waves and assumes the character of a gas of particles with the solitons constituting the particles of different speeds, momenta and energies [88-89].

We have studied in chapter 2 the propagation of electromagnetic waves through a cold collision free plasma in (1+1) and in (2+1) dimensions [sections **2.3**,**2.5**]. In this chapter we study the interaction among non-resonant solitary waves in a cold collisionless plasma with different group velocities that are not close to each other using an asymptotic perturbation method, based on Fourier expansion and spatio-temporal scaling. The usual behaviour of soliton interactions are not to be expected here [90-93] but here we are interested in the interaction of solitary wave solutions of two different nonlinear evolution equations describing two different modes of propagation of waves through the system. Using the multiscale expansion method [90] it has been shown that the interaction system is not described by a nonlinear evolution equation of the standard type but by a system of linear hyperbolic equations.

In chapter 2 we have considered the case of wave propagation with phase velocity V_0. In the next we consider the propagation mode with phase velocity V_1.

4.2 The propagation mode with velocity V_1

The dispersion relation has two branches with a finite slope V=$\frac{\omega}{k}$ at the origin. For one of these branches we have just seen in chapter 2 that in the long -wave approximation, the waves are governed by mKdV equation. Now we will study the behaviour of the second branch, i.e., the mode of propagation with velocity V_1. Proceeding as in section **2.3** we find

At order ε^0

$$S^{(0)} \times E^{(0)} = 0 \tag{4.1}$$

$$\frac{\partial^2 (E_x^{(0)} + S_x^{(0)})}{\partial \xi^2} = 0 \tag{4.2}$$

At order ε^1

from Eq.(2.11) we can get:

$$2V^2 \frac{\partial S^{(0)}}{\partial \xi} = S^{(0)} \times (E^{(1)} - \alpha S^{(1)}) \tag{4.3}$$

From Eq.(4.1) it is found that $E^{(0)}$ and $S^{(0)}$ are collinear vectors. So we can write $E^{(0)} = \alpha S^{(0)}$, and $S^{(0)}$ =s and $E^{(0)} = \alpha s$, constant vectors characterizing the initial static state of the system. Using this fact in Eq.(4.3) we can find that $(E^{(1)} - \alpha S^{(1)})$ and s are collinear vectors. Let us now define g(ξ, τ) as

$$(E^{(1)} - \alpha S^{(1)}) = (1 + \alpha)\beta g(\xi, \tau)s. \tag{4.4}$$

At the same order Eq.(2.8) gives components as:

$$\frac{\partial^2 (E_x^{(1)} + S_x^{(1)})}{\partial \xi^2} = 0 \tag{4.5}$$

$$V^2 \frac{\partial^2}{\partial \xi^2} (\gamma E_y^{(1)} + S_y^{(1)}) = 0, \tag{4.6}$$

$$V^2 \frac{\partial^2}{\partial \xi^2} (\gamma E_z^{(1)} + S_z^{(1)}) = 0, \tag{4.7}$$

In the present case we intend to find the time evolution of the propagation mode with velocity V_1. The scaling that we choose must be consistent with the homogeneity properties of this system. To proceed further, we choose a scale transformation

$$\hat{g} = \varepsilon^2 g,$$

$$\xi = \varepsilon(x - Vt) \tag{4.8}$$

$$\tau = \varepsilon^3 t$$

This will yield

$$E^{(1)} = 0 \quad and \quad S^{(1)} = 0.$$

At order ε^2

from Eq.(2.11) we get

$$2V^2 \frac{\partial S^0}{\partial \xi} = S^0 \times (E_m^{(2)} - \alpha S_m^{(2)}), \qquad m = x, y, z \tag{4.9}$$

$S^{(0)}$ =s and $E^{(0)} = \alpha s$ are constant vectors characterizing the initial static state of the system. Using this fact in Eq.(4.9) we can find that $(E^{(2)} - \alpha S^{(2)})$ and s are collinear vectors. So we can write

$$(E^{(2)} - \alpha S^{(2)}) = (1+\alpha)\beta g(\xi,\tau)s, \tag{4.10}$$

where $g(\xi,\tau)$is an arbitrary function. At the same order Eq.(2.8) gives components as:

$$\frac{\partial^2}{\partial \xi^2}(E_x^{(2)} + S_x^{(2)}) = 0 \tag{4.11}$$

$$\frac{\partial^2}{\partial \xi^2}(\gamma E_y^{(2)} + S_y^{(2)}) = 0, \tag{4.12}$$

$$\frac{\partial^2}{\partial \xi^2}(\gamma E_z^{(2)} + S_z^{(2)}) = 0, \tag{4.13}$$

From Eqs.(4.11-4.13) we get

$$S_x^{(2)} = -E_x^{(2)} \tag{4.14}$$

$$S_y^{(2)} = -\gamma E_y^{(2)} \tag{4.15}$$

$$S_z^{(2)} = -\gamma E_z^{(2)} \tag{4.16}$$

Substituting Eqs.(4.14-4.16) in Eq.(4.10), we get the components of $S^{(2)}$ and $E^{(2)}$ as

$$S^{(2)} = s^{(2)}g \tag{4.17}$$

$$E^{(2)} = e^{(2)}g \tag{4.18}$$

where

$$s^{(2)} = \begin{pmatrix} \beta s_x \\ -\gamma(1+\alpha)s_y \\ 0 \end{pmatrix} \tag{4.19}$$

$$e^{(2)} = \begin{pmatrix} \beta s_x \\ (1+\alpha)s_y \\ 0 \end{pmatrix} \tag{4.20}$$

where γ and β are given by Eqs.(2.31-2.35).

At order ε^2

substituting Eqs.(4.14–4.16) in Eq.(4.9) we get

$$S_y^0 E_z^{(2)}(1+\gamma\alpha) = 0 \tag{4.21}$$

$$S_x^0 E_z^{(2)}(1+\gamma\alpha) = 0 \tag{4.22}$$

$$S_y^0 E_x^{(2)}(1+\alpha) - S_x^0 E_y^{(2)}(1+\gamma\alpha) = 0 \tag{4.23}$$

The system of Eqs.(4.21-4.23) will have a nontrivial solution only if the determinant of the augmented matrix is zero. This gives

$$\beta s_0^2 \cos^2\phi + \gamma(1+\alpha)s_0^2 \sin^2\phi = 0 \tag{4.24}$$

Using expressions for γ and β from Eqs.(2.31-2.34) we obtain the velocity in this case as:

$$V_1 = \sqrt{\frac{\alpha + \sin^2\phi}{1+\alpha}}c. \tag{4.25}$$

and now we can identify V as V_1. At order ε^3 :the expressions for $E^{(3)}$ and $S^{(3)}$ becomes

$$S^{(3)} = \gamma V_1 (s_x/s_y) e_z \frac{\partial g}{\partial \xi} \tag{4.26}$$

$$E^{(3)} = -V_1 (s_x/s_y) e_z \frac{\partial g}{\partial \xi} \tag{4.27}$$

we call e_x, e_y, e_z the vectors of the reference frame

At order ε^4 the compatibility condition is trivial and we find

$$S_x^{(4)} = -E_x^{(4)} = -\beta m_x \tag{4.28}$$

$$S_y^{(4)} = -\gamma E_y^{(4)} + \frac{2(1+\alpha)s_y}{V_1^3} \int_{-\infty}^{\xi} \partial_\tau g \tag{4.29}$$

$$E_y^{(4)} = \frac{\gamma V_1^2}{\mu s_y} \frac{\partial^2 g}{\partial \xi^2} + \frac{2(1+\alpha)\alpha s_y}{V_1^3 \mu} \int_{-\infty}^{\xi} \partial_\tau g \tag{4.30}$$

$$S_z^{(4)} = \gamma V_1 \frac{s_x}{s_y} \partial_\xi g \tag{4.31}$$

where ∂_τ stands for $\frac{\partial}{\partial_\tau}$ and ∂_ξ stands for $\frac{\partial}{\partial_\xi}$.

At order ε^5 Eq.(2.11) reads as:

$$-V_1 \partial_\xi S^{(4)} + \partial_\tau S^{(2)} = -(S^{(0)} \times E^{(5)} + S^{(2)} \times E^{(3)} + +S^{(3)} \times E^{(2)} + S^{(5)} \times E^{(0)} \tag{4.32}$$

Taking the dot product of Eq.(4.32) with $S^{(0)}$ we obtain the compatibility condition

$$-V_1 \partial_\xi s.S^{(4)} + \partial_\tau s.S^{(2)} = s.(S^{(2)} \times E^{(3)} + S^{(3)} \times E^{(2)}) \tag{4.33}$$

Using the results of previous orders we reduce the above equation to the KdV equation:

$$\partial_\tau g + A g \partial_\xi g + B \partial_\xi^3 g = 0 \tag{4.34}$$

where

$$A = \frac{\beta V_1}{2\alpha}, \qquad B = \frac{\gamma V_1^5}{2\alpha(1+\alpha)s_y^2}$$

The KdV equation describes weakly nonlinear dispersive waves in various branches of physics. This equation is completely integrable by the IST method [19], and admits soliton solutions of the form:

$$g = \frac{12k^2 B}{A} \sec h^2 k(\xi - 4k^2 B\tau) \tag{4.35}$$

where k is an arbitrary constant

4.3 The interaction system in (1+1) dimensions

In this section, we derive the equations that govern the interaction between the solitary wave solutions corresponding to two different modes of propagation of the waves through plasma. The KdV equation with speed $\sqrt{\frac{\alpha+\sin^2\phi}{1+\alpha}}c$ appears with a very low amplitude scale while mKdV equation is obtained for high intensity waves with speed $\sqrt{\frac{\alpha}{1+\alpha}}c$. The self interaction of one wave occurs at one scale, while the self interaction of the other wave occurs at a second scale. In order to study the interaction between these two waves moving with different group velocities and amplitudes, we must consider time and space scales of the same order. Therefore we use a third scale to study the interaction of two waves. Let us choose a different scale for the space -time variables as

$$\xi = \varepsilon^2 x \tag{4.36}$$

$$\tau = \varepsilon^2 t \tag{4.37}$$

Let us solve the system by substituting the above scale transformations and the perturbation expansions of the dependent variables given by Eqs.(2.22-2.23) in the Eqs.(2.8) and (2.11) and collecting coefficients of different orders of ε. Thus we find:

At order ε^0

$$\partial_\tau^2 (E_m^{(0)} + S_m^{(0)}) = \partial_\xi^2 E_m^{(0)} \tag{4.38}$$

$$E_x^{(0)} + S_x^{(0)} = (1+\alpha) s_x \tag{4.39}$$

$$E^{(0)} = \alpha S^{(0)} \tag{4.40}$$

$$(\partial_\tau^2 - V_0^2 \partial_\xi^2) S_x^{(0)} = 0 \tag{4.41}$$

where

$$V_0 = \sqrt{\frac{\alpha}{1+\alpha}} c.$$

As in the previous case $S_x^{(0)}$ is a constant. $S_y^{(0)}$ and $S_z^{(0)}$ may propagate with the speed V_0.

At order ε^1

from Eq.(2.11), we get

$$S^{(0)} \times (E_m^{(1)} - \alpha S_m^{(1)}) = 0 \tag{4.42}$$

We now introduce a function f=f(ξ, τ) such that

$$(E_m^{(1)} - \alpha S_m^{(1)}) = f(\xi, \tau) S^{(0)}$$

i.e,

$$S_m^{(1)} = \frac{1}{\alpha} E_m^{(1)} - \frac{f}{\alpha} S^{(0)} \tag{4.43}$$

From Eq.(2.8) we get

$$\frac{\partial^2}{\partial \tau^2} E_m^{(1)} - V_0^2 \frac{\partial^2}{\partial \xi^2} E_m^{(1)} = \frac{1}{1+\alpha} \frac{\partial^2}{\partial \tau^2} S_m^{(0)}, \ m = y, z \tag{4.44}$$

$$E_x^{(1)} = -S_x^{(1)} = \frac{s_x}{1+\alpha} f \tag{4.45}$$

At order ε^2

$$\partial_\tau S^{(0)} = -S^{(0)} \times [E^{(2)} - \alpha S^{(2)}] - S^{(1)} \times E^{(1)} \tag{4.46}$$

$$\partial_\tau^2 (E_x^{(2)} + S_x^{(2)}) = 0 \tag{4.47}$$

$$\partial_\tau^2 (E_m^{(2)} + S_m^{(2)}) = \partial_\xi^2 E_m^{(2)} \tag{4.48}$$

We introduce a function $\theta(\xi, \tau)$ such that $\theta \to 0$ as $\xi \to -\infty$ and then we can write

$$S_y^{(0)} = A \cos\theta, \quad S_z^{(0)} = A \sin\theta, \qquad A = S_0^{(0)} \sin\phi \tag{4.49}$$

Putting

$$S_{\perp}^{(k)} = S_y^{(k)} + iS_z^{(k)} \tag{4.50}$$

$$E_{\perp}^{(k)} = E_y^{(k)} + iE_z^{(k)} \tag{4.51}$$

$$S_{\perp}^{(1)} = \frac{1}{\alpha}E_{\perp}^{(1)} - \frac{s_y}{\alpha}fe^{i\theta} \tag{4.52}$$

$$\frac{\partial^2}{\partial\tau^2}E_{\perp}^{(1)} - V_0^2\frac{\partial^2}{\partial\xi^2}E_{\perp}^{(1)} = \frac{s_y}{1+\alpha}\frac{\partial^2}{\partial\tau^2}fe^{i\theta} \tag{4.53}$$

Substituting Eq.(4.43) in Eq. (4.46) and using Eq.(4.49) we get

$$S^{(0)} \times (-\partial_\tau\theta + [E^{(2)} - \alpha S^{(2)}] - \frac{f}{\alpha}E^{(1)}) = 0 \tag{4.54}$$

So we can introduce $h(\xi,\tau)$ such that

$$\partial_\tau\theta + [E^{(2)} - \alpha S^{(2)}] - \frac{f}{\alpha}E^{(1)}) = h(\xi,\tau)s$$

We have from Eq.(4.47)

$$E_x^{(2)} = -S_x^{(2)}$$

$$\frac{\partial^2}{\partial\tau^2}E_{\perp}^{(1)} - V_0^2\frac{\partial^2}{\partial\xi^2}E_{\perp}^{(1)} = \frac{\partial^2}{\partial\tau^2}(\frac{\alpha}{1+\alpha}fE_{\perp}^{(1)} + \frac{s_y}{1+\alpha}he^{i\theta}) \tag{4.55}$$

At order ε^3

we have the equation

$$\partial_\tau S^{(1)} = -S^0 \times [E^{(3)} - \alpha S^{(3)}] - [S^{(1)} \times E^{(2)} + S^{(2)} \times E^{(1)}] \tag{4.56}$$

Taking dot product of Eq.(4.56) with $S^{(0)}$ and after some algebra, we obtain

$$\frac{\alpha s_x^2 + (1+\alpha)s_y^2}{\alpha(1+\alpha)}\partial_\tau f = \frac{S_y}{\alpha}[(\partial_\tau \theta)(sin\theta E_y^{(1)} - cos\theta E_z^{(1)}) - (\cos\theta \partial_\tau (E_y^{(1)}) + \sin\theta \partial_\tau (E_z^{(1)}))] \tag{4.57}$$

We now define two quantities ψ and ϕ

$$\psi = sin\theta E_z^{(1)} + cos\theta E_y^{(1)} \tag{4.58}$$

$$\phi = cos\theta E_z^{(1)} - sin\theta E_y^{(1)} \tag{4.59}$$

Equation (4.57) can be integrated once to yield

$$f = \frac{s_y}{s^2 V_1^2}\psi \tag{4.60}$$

where

$$V_1 = \sqrt{\frac{\alpha + \sin^2\varphi}{1+\alpha}}c \tag{4.61}$$

and

$$s^2 = (S_x^{(0)})^2 + (S_y^{(0)})^2 \tag{4.62}$$

and V_1 is the velocity the KdV mode. Using the definitions given by Eqs.(4.58-4.60), Eq.(4.53) can be written as

$$[\partial_\tau^2 - V_0^2 \partial\zeta^2]\Sigma \exp i\theta = (1 - \frac{V_0^2}{V_1^2})\partial_\tau^2 \psi \exp i\theta \tag{4.63}$$

where

$$\Sigma = \psi + i\phi \tag{4.64}$$

This can be reduced to the system of two real equations given as:

$$[\partial_\xi^2 - \frac{1}{V_0^2}\partial_\tau^2]\phi = [-2\theta'(\partial_\xi + \frac{V_0}{V_1^2}\partial_\tau) + (1 - \frac{V_0^2}{V_1^2})\theta'']\psi \tag{4.65}$$

$$[\partial_\xi^2 - \frac{1}{V_1^2}\partial_\tau^2 - (1 - \frac{V_0^2}{V_1^2})\theta'^2]\psi = [-2\theta'(\partial_\xi + \frac{1}{V_0}\partial_\tau)]\phi \tag{4.66}$$

Equations (4.65) and (4.66) are linear hyperbolic equations describing the interacting system.

4.4 (2+1) dimensional case:

In this case there exists a weak dependence of the field variables on the y-axis and this can represent a kind of transversal perturbation, so that we introduce the following scaling transformations

$$\xi = \varepsilon(x - Vt) \tag{4.67}$$

$$\zeta = \varepsilon^2 y \tag{4.68}$$

$$\tau = \varepsilon^3 t. \tag{4.69}$$

Proceeding as in (1+1) dimensional case for the wave with the velocity V_0, that is, giving a perturbative expansion for the field variables and equating the coefficients of different orders of ε. At order ε^2 the system of equations can be reduced to a modified form of Kadomtsev-Petviashvili equation (mKP).

$$[\frac{\partial f}{\partial \tau} + \frac{3}{2}\mu f^2 \frac{\partial f}{\partial \xi} + \mu \frac{\partial^3 f}{\partial \xi^3}]_\xi = \sigma \frac{\partial^2 f}{\partial \zeta^2} \tag{4.70}$$

The steady state solution of this equation is found to be

$$f = -2k^2 sech^2 k\eta, \qquad \eta = \xi + \zeta - u\tau,$$

u is a constant, $k^2 = \frac{u-\sigma}{\mu}$

Proceeding as in the case of (1+1) dimensional case, for the wave with the velocity V_1 we get at order ε^4:

$$E_y^{(4)} = \frac{\gamma V^2}{\mu s_y}\frac{\partial^2 g}{\partial \xi^2} + \frac{2(1+\alpha)\alpha s_y}{V^3 \mu}\int_{-\infty}^{\xi} \partial_\tau g + \int_{-\infty}^{\xi}\int_{-\infty}^{\xi} \frac{\alpha(1+\alpha)}{\mu s_y}\frac{\partial^2 g}{\partial \zeta^2}(d\xi)^2 \tag{4.71}$$

$$E_z^{(4)} = \frac{\gamma V_1 s_x}{s_y \partial_\xi g} \tag{4.72}$$

At order ε^5

Eq.(4.11) becomes

$$-V_1 \partial_\xi S^{(4)} + \partial_\tau S^{(2)} = -(S_0 \times E^{(5)} + S^{(2)} \times E^{(3)} + +S^{(3)} \times E^{(2)} + S^{(5)} \times E_0) \tag{4.73}$$

with the same assumptions as in the case of (1+1) dimensions we obtain the Kadomtsev-Petviashvili (KP) equation in (2+1) dimensions

$$[\frac{\partial f}{\partial \tau} + \frac{3}{2}\mu f\frac{\partial f}{\partial \xi} + \mu\frac{\partial^3 f}{\partial \xi^3}]_\xi = \sigma\frac{\partial^2 f}{\partial \zeta^2} \tag{4.74}$$

The soliton solution of this equation is found to be

$$f = -2a^2 sech(a\eta)$$

where $\eta = \xi + \zeta - u\tau$, u is a constant, $a^2 = (\frac{u-\sigma}{\mu})$.

The interaction system in (2+1) dimensions is studied by choosing a different scale transformations

$$\xi = \varepsilon^2(x - Vt) \tag{4.75}$$

$$\zeta = \varepsilon^2 y \tag{4.76}$$

$$\tau = \varepsilon^2 t. \tag{4.77}$$

By following the similar procedures that we have used for the derivation of the interaction system in (1+1) dimensions, in (2+1) dimensions the interacting system can be described with the following set of equations

$$[\partial_\xi^2 + \partial_\zeta^2 - \frac{1}{V_0^2}\partial_\tau^2]\phi = [-2\theta'(\partial_\xi + \partial_\zeta + \frac{V_0}{V_1^2}\partial_\tau) + 2(1 - \frac{V_0^2}{V_1^2})\theta'']\psi \tag{4.78}$$

$$[\partial_\xi^2 + \partial_\zeta^2 - \frac{1}{V_1^2}\partial_\tau^2 - 2(1 - \frac{V_0^2}{V_1^2})\theta'^2]\psi = [-2\theta'(\partial_\xi + \partial_\zeta + \frac{1}{V_0}\partial_\tau)]\phi \tag{4.79}$$

4.5 Analysis of the interaction system in the (1+1) dimensional case:

We can see from Eqs.(4.65-4.66) that ϕ propagates at velocity V_0, thus belongs to the mKdV mode while ψ propagates with velocity V_1 this representing the KdV mode. This feature can be verified by putting θ constant in Eqs. (4.65) and (4.66). Then we have

$$[\partial_\xi^2 - \frac{1}{V_1^2}\partial_\tau^2]\psi = 0 \tag{4.80}$$

$$[\partial_\xi^2 - \frac{1}{V_0^2}\partial_\tau^2]\phi = 0 \tag{4.81}$$

Here we are interested in a particular solution that is

$$\psi = \psi(\xi - V_1\tau) \tag{4.82}$$

$$\phi = 0 \tag{4.83}$$

Then using Eqs.(4.58-60) and Eqs.(4.43, 4. 45), we find

$$E = \frac{s_x s_y}{(1+\alpha)s^2V_1^2}\psi + \psi \tag{4.84}$$

Equation (4.84) is identical to Eq.(4.20), gives the same expressions, in the frame where we derived the KdV equation for the function g with the identification: $\beta = \frac{\sin^2\phi}{V_1^2}$ and g=$\frac{1}{(1+\alpha)s_y}\psi$. Thus we can show that ψ represents the KdV mode.

Let us now show that ϕ can be a correction term to θ. We now look for solutions to the Eqs.(4.65) and (4.66) for which ψ =0. Then we have

$$\theta'(\partial_\xi + \frac{1}{V_0}\partial_\tau)\ \phi = 0 \tag{4.85}$$

and

$$[\partial_\xi^2 - \frac{1}{V_0^2}\partial_\tau^2]\phi = 0 \tag{4.86}$$

We obtain a solution with

$$\phi = \phi(\xi - V_0\tau). \tag{4.87}$$

Under this condition, from Eqs.(4.43-4.45) and from Eqs.(4.58-4.60), we can write

$$E_x^{(1)} = 0 \quad because\ \psi = 0$$

$$E_y^{(1)} = -\phi\sin\theta \ \ and\ \ E_z^{(1)} = \phi\cos\theta \tag{4.88}$$

Let

$$E_\perp^{(1)} = E_y^{(1)} + iE_z^{(1)}, \tag{4.89}$$

$$E_\perp = E_\perp^{(0)} + \varepsilon E_\perp^{(1)} + O(\varepsilon^2). \tag{4.90}$$

Using Eqs.(2.73) and (4.88), Eq.(4.90) can be written as

$$E_\perp = s_y \exp i\theta(1 + i\varepsilon\frac{\phi}{s_y} + O(\varepsilon^2)) \tag{4.91}$$

We can say that $\varepsilon\frac{\phi}{s_y}$ is the first order correction to θ-wave. Equations (4.65) and (4.66) describe the interaction between the two waves ψ and $\theta + \varepsilon\frac{\phi}{s_y}$. From Eq.(91) we can say that the interaction is by itself nonlinear but in the present study we have taken θ as given data, and have dropped all orders higher than $\varepsilon \frac{\psi}{s_y}$ and therefore the obtained system is linearized.

We can now analyze this interaction system using the ansatz on the shape of the ψ and θ waves. θ describes the mKdV mode and therefore θ' should look like a soliton of mKdV as described in section **2.3**. It has a bell shape, and vanishes quickly at both + ∞ and -∞ in time and space. In the same way, ψ should look like a soliton of KdV equation given by Eq.(4.34). In order to perform an explicit calculation, it seems useful to model the bell shape of these waves, such as delta functions.This would describe an asymptotic case, as the typical width of the solitons is very small in regard to the space scale at which the interaction occurs. We can show this using scalings. Consider two given input solitary waves. A scale parameter ε_0 is defined by their amplitude. The mKdV equation concerns a first order quantity, thus for the mKdV, the parameter ε in Eq.(2.12) is ε_0 and the typical width of a soliton of this mode is

$$L_{mkdv} = \frac{L}{\varepsilon_0}$$

L being a length of order unity. The KdV equation concerns a second order quantity, thus in this case $\varepsilon = \sqrt{\varepsilon_0}$ and the typical length of the KdV soliton is

$$L_{kdv} = \frac{L}{\sqrt{\varepsilon_0}}.$$

The interaction system given by Eqs.(4.65) and (4.66) concerns first order quantities, but the scaling is different, and the typical width of the interaction process is given by

$$L_{int} = \frac{L}{\varepsilon_0^2}.$$

Therefore, while $\varepsilon_0 << 1$, $L_{int} << L_{mKdV} << L_{KdV}$. This shows that the width of the ϕ wave i.e., mKdV soliton is much less than that of the ψ wave corresponding to the KdV soliton. Therefore, it is justified to assume that the ψ wave will be a pulse of large width containing a large number of solitons of smaller widths and the effect of the ψ wave will be an averaged effect of the smaller pulses. Hence the soliton-soliton interaction of the original system reduces to that of a soliton -wave interaction. It has been found that the analysis described above can be extended to study the solitary wave interactions in (2+1) dimensions and similar conclusions could be achieved at.

4.6 Conclusion

The well studied type of interaction between soliton solutions of the same equation is of resonant type which can lead to only a phase change of the solitons. In the present study we have not considered soliton-soliton interaction of the same nonlinear evolution equations and therefore the usual behaviour of soliton-soliton interaction is need not to be expected here.

We also found that the wave evolves into a chain of solitons of different amplitudes, widths and speeds. The interaction system has been analyzed in the particular case where the duration of the incident pulse of mKdV mode is very short with respect to the interaction time (or space) scale. This particular case corresponds to a one -solitonic solution of the mKdV equation but disregards the solitonic structure of the wave of the KdV mode. The strong plasma turbulence is characterised by waves in plasmas evolving into a chain of solitons with different amplitude, widths and speeds. The present study deals with such a system and the interaction of the solitons may throw more light into the study of plasma turbulence and the studies may require both analytical and numerical methods.

Chapter 5

Nonlinear wave propagation through a ferromagnet in (2+1) dimensions with damping

It often happens that the requirements of
simplicity and beauty are the same,
but where they clash,
the latter must take precedence.

P A. M. Dirac

5.1 Introduction

The problem of propagation of electromagnetic waves in ferromagnetic dielectrics is a very complex matter that has given rise to many studies. The phenomenon of propagation of electromagnetic waves in ferromagnets are not only interesting in itself but are also important in connection with the behaviour of ferrite devices at microwave frequencies such as ferrite loaded waveguides, magneto-optical recording systems, etc. The concept of magneto-optical recording has become technologically very important for the purpose of high storage and fast reading. Electromagnetic waves in ferromagnets have been investigated by several authors so far [82-90]. Wave propagation in ferromagnetic media has received new interest for several reasons: on the one hand, the so-called Maxwell-Landau model governing these phenomena is of major interest for mathematicians due to its nonlinear properties, and thus also for the theory of nonlinear wave propagation, from the point of view of theoretical physics. The propagation of electromagnetic waves is obviously described by the Maxwell equations, with some constitutive relations characterizing the medium in which the wave propagates [99-100].

The propagation of electromagnetic waves in a ferromagnet obeys nonlinear equations with dispersion and dissipation. It is well known that when we consider the weak nonlinear case it is possible to reduce a large class of 1-D nonlinear systems to soliton systems governed by KdV equation, mKdV and the nonlinear Schrodinger equations. Recently several authors [91-98, 101-105] gave a rigorous study of the propagation of long wave length electromagnetic waves in a saturated ferromagnet taking into account of nonlinearity and dissipation in (1+1) dimensions.

In this chapter we investigate how dissipation and nonlinearity can affect the electromagnetic waves propagating through a saturated ferromagnet in the presence of an external magnetic field in (2+1) dimensions. The model assumes that the medium is immersed in a constant external magnetic field strong enough to magnetize it to saturation. By using the long wave approximation of the reductive perturbation method (RPM) [39], it is found that the system of equations can be reduced to an integro-differential equation

5.2 Formulation of the problem

The linear study of plane waves, which can be found in [108], uses a simple model assuming that the medium is infinite and neglecting the inhomogeneous exchange interaction, anisotropy, and damping. The strong correlation between the directions of atomic spins in magnetically ordered crystals leads to the exixtence of a particular type of collective mode in such crystals. To understand the origin of these modes we consider a ferromagnet at $T = 0$. All the atomic magnetic moments then have the same direction which corresponds to minimum energy of the ferromagnet. The magnetic moment gets deflected and this change of direction will not remain localized but will be propagated in the form of waves through the ferromagnet due to exchange interaction.

To investigate these waves it will be necessary to establish the time dependence of the magnetic moment density. The motion of a magnetic moment M in an external magnetic field H, is governed by the "torque" equation. The damping of these waves is due to the interaction with each other and also with lattice vibrations and conduction electrons. We are considering electromagnetic waves in a ferromagnet under an external magnetic field in the presence of dissipative effect in (2+1) dimensions. In ferromagnetic media, the dependency of the dielectric polarizability with regard to the electric field is linear and scalar, while the magnetic moment $\vec{M}$ and induction $\vec{B} = \mu_0(\vec{H} + \vec{M})$ are coupled through the so-called Landau equation So the basic equations relevant to the present problem are the following

$$\nabla \times \vec{E} = \frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \quad \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \tag{5.1}$$

The magnetic induction $\vec{B}$ with magnetic field $\vec{H}$ are related to the magnetization of the medium through the relation:

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

where μ_0 is the magnetic permeability in vacuum The magnetization density M is governed by the "torque" equation:

$$\frac{\partial \vec{M}}{\partial t} = -\mu_0 \gamma \vec{M} \times \vec{H} - \delta \frac{\vec{M} \times (\vec{M} \times \vec{H})}{M^2} \tag{5.2}$$

where γ is the gyromagnetic ratio and δ is a positive constant. Following Landau and Lifshitz [99-100], we have assumed that the dissipative effect in the ferromagnet is incorporated by taking into account the second term on the right hand side of Eq.(5.2). Taking the time derivative of Eq.(5.1) and then substituting for B and E we can get,

$$-\nabla(\nabla.H) + \nabla^2 \vec{H} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\vec{H} + \vec{M}) \tag{5.3}$$

where c=$\frac{1}{\sqrt{\mu_0 \epsilon}}$ is the speed of light based on the dielectric constant of the ferromagnet. Equations (5.2) and (5.3) are simultaneous equations for $\vec{M}$ and $\vec{H}$

Let us consider the propagation of plane waves in (2+1) dimensions. All the physical quantities are assumed to be functions of two space coordinates x, y and the time coordinate t. Let us seek a solution of these equations in the form of a Fourier expansion in harmonics of the fundamental $\tilde{E}$=exp $i(kx - \omega t)$ as,

$$M = \sum_{n=-\infty}^{+\infty} M^n \tilde{E}^n, \quad H = \sum_{n=-\infty}^{+\infty} H^n \tilde{E}^n \tag{5.4}$$

We assume here that the ferromagnet is in a uniform state. Writing $\vec{M}$=(M_x,M_y,M_z) and $\vec{H}$=(H_x,H_y,H_z) and expressing the Fourier components of M and H in powers of a small parameter ε we can write:

$$M^n = \sum_{j=0}^{\infty} \varepsilon^j M_j^n(x, y, t)$$

$$H^n = \sum_{j=o}^{\infty} \varepsilon^j H_j^n(x, y, t). \tag{5.5}$$

$\vec{M}$ and $\vec{H}$ satisfy the following boundary conditions

$$M_{jx} \to 0, \quad except\, that \;\; M_{0x} \to m_0 cos\phi = m_{0x} \tag{5.6}$$

$$M_{jy} \to 0, \quad except\, that \;\; M_{0y} \to m_0 sin\phi = m_{0y} \tag{5.7}$$

$$M_{jz} \to 0 \tag{5.8}$$

as $\xi \to -\infty$.

$$H_{jx} \to 0, \quad except\, that \;\; H_{0x} \to h_0 cos\phi \tag{5.9}$$

$$H_{jy} \to 0, \quad except\, that \;\; H_{0y} \to h_0 sin\phi \tag{5.10}$$

$$H_{jz} \to 0 \tag{5.11}$$

as $\xi \to -\infty$.

where m_0, h_0 and ϕ are positive constants and $0 \leq \phi \leq \frac{\pi}{2}$.

5.3 Dispersion relation

Before proceeding to the nonlinear problem, we will first examine the dispersion relation in the linearized limit. Assuming a sinusoidal wave form for the field variables given by Eq.(5.4), Eqs.(5.2) and (5.3) can be cast in the form as:

$$[\frac{\partial}{\partial t} - in\omega]M^n = -\mu_0\gamma[\sum_{p+q=n} M_j^p \times H_j^q] - \delta[\sum_{p+q+r=n} M_j^p \times [M_j^q \times (H_j^r - \alpha M_j^r)] \quad (5.12)$$

$$[\frac{\partial^2}{\partial t^2} + 2in\omega\frac{\partial}{\partial t} - n^2\omega^2][M_{js}^n + H_s^n] = c^2[\frac{\partial^2}{\partial x^2} + 2ink\frac{\partial}{\partial x} - n^2k^2]H_{js}^n(1-\delta_{sx}) \quad (5.13)$$

where $s = x, y, z$ and δ_{sx} is the Kronecker delta function. For j=0, we find that H_0^0 and M_0^0 are collinear vectors and thus we define α such that $H_0 = h_0 = \alpha M_0^0 = \alpha m_0$, $m_0 = (m_x, m_y, o) = (m_0 \cos\phi, m_0 \sin\phi, 0)$. Taking the leading order terms for the order (n=1) from both equations, we can write the components of M_s^1 as functions of H_s^1. Thus we can find a linear homogeneous system of equations for H_x^1, H_y^1 and H_z^1

$$in\omega H_{1x}^n + M_{0y}\beta H_{1z}^n = 0 \quad (5.14)$$

$$in\omega\gamma H_{1y}^n - M_{0x}\beta H_{1z}^n = 0 \quad (5.15)$$

$$-(1+\alpha)M_{0y}H_{1x}^n + M_{0x}\beta E_{1y}^n + in\omega\gamma H_{1z}^n = 0. \quad (5.16)$$

The system will have a non-trivial solution only if the determinant of the augmented matrix is zero. The determinant of this system of equations, $\Delta(n)$, is

$$\Delta(n) = in\omega[-n^2\gamma^2\omega^2 + \beta^2(m_{0x})^2 + \gamma\beta(1+\alpha)(m_{0y})^2] \quad (5.17)$$

where

$$\beta = (1 + \alpha\gamma) \quad (5.18)$$

and

$$\gamma = (1 - \frac{c^2k^2}{\omega^2}), \tag{5.19}$$

For n=1, $\Delta(1)$ is zero if ω satisfies the dispersion relation

$$-\gamma^2\omega^2 + \beta^2(m_0)^2\cos^2\phi + \gamma\beta(1+\alpha)(m_0)^2\sin^2\phi = 0 \tag{5.20}$$

Writing γ and β in terms of α, k and ω we can obtain an equation which is cubic in ω^2

$$[1+\alpha(1-\frac{c^2k^2}{\omega^2})]^2(m_{0x})^2+[1-\frac{c^2k^2}{\omega^2}][1+\alpha(1-\frac{c^2k^2}{\omega^2})](1+\alpha)(m_{0y})^2 = \omega^2(1-\frac{c^2k^2}{\omega^2}) \tag{5.21}$$

This equation is similar to the dispersion relation given by Eq.(2.39). Here also there can be two types of wave propagation through the ferromagnets with phase velocities given by

$$V_0 = \sqrt{\frac{\alpha}{1+\alpha}}c \tag{5.22}$$

$$V_1 = \sqrt{\frac{\alpha+\sin^2\phi}{1+\alpha}}c \tag{5.23}$$

We assume that the y-co-ordinate is orthogonal to the plane determined by the external magnetic field H^0 and the direction of propagation of the perturbation is along the x-axis. $M^0 = m_0$ and $H^0 = \alpha m_0$ are constant vectors characterizing the initial static state of the system.

We will now introduce the stretching variables ξ, ζ and τ as,

$$\xi = \varepsilon(x - Vt) \tag{5.24}$$

$$\zeta = \varepsilon^2 y \tag{5.25}$$

$$\tau = \varepsilon^3 t \tag{5.26}$$

where ε is the small parameter measuring the weakness of dispersive effect. The velocity V is obtained from Eq.(5.22). The expression for ζ represents weak dependence of the field parameters on the coordinate y.

Applying the scale transformations given by Eqs.(5.24-26) and substituting the expansions given by Eqs.(5.5) in (5.2) and (5.3) and then collecting and solving coefficients of different orders of ϵ^j for $n = 1$ we get at order ε^0

$$-\mu_0\gamma[M_{0y}H_{0z} - M_{0z}H_{0y}] - \delta[(1+\alpha)M_{0y}M_{0x} - (1+\alpha)M_{0z}M_{0x}] = 0 \tag{5.27}$$

$$-\mu_0\gamma[M_{0z}H_{0x} - M_{0x}H_{0z}] - \delta[(1+\alpha)M_{0z}M_{0y} - (1+\alpha)M_{0x}M_{0y}] = 0 \tag{5.28}$$

$$- \mu_0\gamma[M_{0x}H_{0y} - M_{0y}H_{0x}] - \delta[(1+\alpha)M_{0z}M_{0x} - (1+\alpha)M_{0z}M_{0y}] = 0 \tag{5.29}$$

$$\frac{\partial^2[H_{0x} + M_{0x}]}{\partial\xi^2} = 0 \tag{5.30}$$

$$V^2\frac{\partial^2}{\partial\xi^2}(\gamma H_{0y} + M_{0y}) = 0, \tag{5.31}$$

at order ε^1

$$V\frac{\partial M_0}{\partial\xi} = \mu_0\gamma_0[m \times (H_1 - \alpha M_1)] - \delta[\frac{\vec{M} \times (\vec{M} \times \vec{H})}{M^2}] \tag{5.32}$$

The y, z components of this equation is

$$- \mu_0\gamma_0(1+\alpha)M_{0z}M_{1x} = V\frac{\partial M_{0y}}{\partial\xi} - \delta[(1+\alpha)(M_{0z})^2M_{1y} - (1+\alpha)(M_{0x})^2M_{1y}] \tag{5.33}$$

$$\mu_0\gamma_0(1+\alpha)M_{0y}M_{1x} = V\frac{\partial M_{0z}}{\partial\xi} - \delta[(1+\alpha)(M_{0x})^2M_{1z} - (1+\alpha)(M_{0y})^2M_{1z}] \tag{5.34}$$

$$H_{1x} + M_{1x} = 0 \tag{5.35}$$

$$\gamma H_{1y} + M_{1y} = 0, \tag{5.36}$$

$$\gamma H_{1z} + M_{1z} = 0, \tag{5.37}$$

$M_0 = m_0$ and $H_0 = \alpha m_0$ are the constant vectors characterizing the initial static state of the system, where $m_0 = (m_x, m_y, 0)$ Using this fact in (5.32) we can show that $H_1 - \alpha M_1$ and m_0 are colinear vectors. Thus we can write

$$H_1 - \alpha M_1 = (1+\alpha)\beta g m_0, \tag{5.38}$$

where $g(\xi, \zeta, \tau)$ is an arbitrary function of ξ, ζ, τ. Substituting Eqs.(5.35-5.37) in Eq.(5.38), we get the first order components given by

$$M_{1x} = \beta m_x g \tag{5.39}$$

$$M_{1y} = \gamma(1+\alpha) m_y g \tag{5.40}$$

$$M_{1z} = 0, \quad since \ m_z = 0$$

at order ε^2

$$-V\frac{\partial M_{1x}}{\partial \xi} = -\mu_0\gamma[M_{0y}(H_{2z} - \alpha M_{2z}) - M_{0z}(H_{2y} - \alpha M_{2y})] - \delta[(1+\alpha)M_{1y}M_{0y}M_{1x} - (1+\alpha)M_{1x}M_{0z}M_{1z}] \tag{5.41}$$

$$-V\frac{\partial M_{1y}}{\partial \xi} = -\mu_0\gamma[M_{0z}(H_{2x} - \alpha M_{2x}) - M_{0x}(H_{2z} - \alpha M_{2z})] - \delta[(1+\alpha)M_{1z}M_{1y}M_{0z} - (1+\alpha)M_{0x}M_{1x}M_{1y}] \tag{5.42}$$

$$-V\frac{\partial M_{1z}}{\partial \xi} = -\mu_0\gamma[M_{0x}(H_{2y} - \alpha M_{2y}) - M_{0y}(H_{2x} - \alpha M_{2x})] - \delta[(1+\alpha)M_{1z}M_{1x}M_{0x} - (1+\alpha)M_{1z}M_{0z}M_{1y}] \tag{5.43}$$

$$(H_{2x} + M_{2x}) = 0 \tag{5.44}$$

$$\frac{\partial(H_{2y} - \alpha M_{2y})}{\partial\xi} = -\frac{2V(1+\alpha)^2}{c^2}\frac{\partial M_{0y}}{\partial\tau} - (1+\alpha)\frac{\partial^2 M_{0y}}{\partial\zeta^2} \tag{5.45}$$

$$\frac{\partial(H_{2z} - \alpha M_{2z})}{\partial\xi} = -\frac{2V(1+\alpha)^2}{c^2}\frac{\partial M_{0z}}{\partial\tau} - (1+\alpha)\frac{\partial^2 M_{0z}}{\partial\zeta^2} \tag{5.46}$$

Solving for H_{2y}, M_{2y} and H_{2z}, M_{2z}, from Eqs.(5.45) and (5.46) we can get

$$(H_{2y} - \alpha M_{2y}) = \int_{-\infty}^{\xi} -\tfrac{2V(1+\alpha)^2}{c^2}\tfrac{\partial M_{0y}}{\partial\tau}d\xi + \int_{-\infty}^{\xi}\int_{-\infty}^{\xi}\alpha(1+\alpha)\tfrac{\partial^2 M_{0y}}{\partial\zeta^2}(d\xi)^2 \tag{5.47}$$

$$(H_{0z} - \alpha M_{0z}) = \int_{-\infty}^{\xi} -\tfrac{2V(1+\alpha)^2}{c^2}\tfrac{\partial M_{0z}}{\partial\tau}d\xi - \int_{-\infty}^{\xi}\int_{-\infty}^{\xi}\alpha(1+\alpha)\tfrac{\partial^2 M_{0z}}{\partial\zeta^2}(d\xi)^2 \tag{5.48}$$

Substituting for

$$(H_{2y} - \alpha M_{2y}) \;\; and \;\; (H_{2z} - \alpha M_{2z}) \tag{5.49}$$

in Eq.(5.41) we get

$$\begin{aligned}\tfrac{V}{\mu_0\gamma}\tfrac{\partial M_{1x}}{\partial\xi} &= M_{0y}[\int_{-\infty}^{\xi}\tfrac{-2V(1+\alpha)^2}{c^2}\tfrac{\partial}{\partial\tau}M_{0z}d\xi + \int_{-\infty}^{\xi}\int_{-\infty}^{\xi}\alpha(1+\alpha)\tfrac{\partial^2 M_{0y}}{\partial\zeta^2}(d\xi)^2] \\ &-M_{0z}\left[\int_{-\infty}^{\xi}\tfrac{-2V(1+\alpha)^2}{c^2}\tfrac{\partial}{\partial\tau}M_{0y}d\xi - \int_{-\infty}^{\xi}\int_{-\infty}^{\xi}\alpha(1+\alpha)\tfrac{\partial^2 M_{0y}}{\partial\zeta^2}(d\xi)^2\right] - \\ &\tfrac{\delta}{\mu_0\gamma}[(1+\alpha)M_{1z}M_{0z}M_{1y} - (1+\alpha)M_{0x}M_{1y}M_{1x}]\end{aligned} \tag{5.50}$$

Introducing two new variables A and $\theta = \theta(\xi, \tau)$ defined by

$$M_{0y} = A\cos\theta, M_{0z} = A\sin\theta, A = m_0 \sin\phi, \theta \to 0 \quad as \;\; \xi \to -\infty. \tag{5.51}$$

Substituting Eq.(5.51) in (5.34), we have

$$M_{1x} = \frac{V}{\mu_0\gamma(1+\alpha)}\frac{\partial\theta}{\partial\xi} \tag{5.52}$$

Now using Eqs.(5.51 – 5.52), Eq.(5.50) can be written as,

$$\begin{aligned} -\mu\tfrac{\partial^2\theta}{\partial\xi^2} &= cos\theta\tfrac{\partial}{\partial\tau}\textstyle\int_{-\infty}^{\xi} sin\theta d\xi + \sigma cos\theta\tfrac{\partial^2}{\partial\zeta^2}\textstyle\int_{-\infty}^{\xi}\int_{-\infty}^{\xi} sin\theta(d\xi)^2 \\ &-sin\theta\tfrac{\partial}{\partial\tau}\textstyle\int_{-\infty}^{\xi} cos\theta d\xi - \sigma sin\theta\tfrac{\partial^2}{\partial\zeta^2}\textstyle\int_{-\infty}^{\xi}\int_{-\infty}^{\xi} cos\theta(d\xi)^2 + \nu cos\theta\tfrac{\partial\theta}{\partial\xi} \end{aligned} \tag{5.53}$$

where

$$\mu = \frac{Vc^2}{2(1+\alpha)^3\mu_0\gamma}$$

$$\sigma = \frac{c^2}{2V(1+\alpha)}$$

$$\nu = \frac{\delta c^2\mu m_0^2 cos\phi sin\phi}{2(1+\alpha)^2\mu_0\gamma}$$

Differentiating Eq.(5.53) with respect to ξ and simplifying we obtain ,

$$\frac{\partial}{\partial\xi}\left[\frac{\frac{\partial\theta}{\partial\tau} + \mu\frac{\partial^3\theta}{\partial\xi^3} - \int_{-\infty}^{\xi}\sigma\frac{\partial^2\theta}{\partial\zeta^2} + \nu cos\theta\frac{\partial^3\theta}{\partial\xi^3}}{\frac{\partial\theta}{\partial\xi}}\right] = -\mu\frac{\partial^2\theta}{\partial\xi^2}[\frac{\partial\theta}{\partial\xi}] - \nu sin\theta[\frac{\partial\theta}{\partial\xi}]^2 \tag{5.54}$$

Eq.(5.54) can be integrated with respect to ξ to give:

$$\frac{\partial\theta}{\partial\tau} + \mu\frac{\partial^3\theta}{\partial\xi^3} - \sigma\int_{-\infty}^{\xi}\frac{\partial^2\theta}{\partial\zeta^2} + \nu\frac{\partial^2\theta}{\partial\xi^2} = -\mu\frac{\partial\theta}{\partial\xi}\int_{-\infty}^{\xi}\frac{\partial^2\theta}{\partial\xi^2}\frac{\partial\theta}{\partial\xi}d\xi - \tag{5.55}$$

$$\nu\frac{\partial\theta}{\partial\xi}\int_{-\infty}^{\xi}[\frac{\partial\theta}{\partial\xi}]^2 \tag{5.56}$$

Putting $f = \frac{\partial\theta}{\partial\eta}$ the above equation becomes

$$\frac{\partial f}{\partial\tau} + \frac{3}{2}\mu f^2\frac{\partial f}{\partial\xi} + \mu\frac{\partial^3 f}{\partial\xi^3} - \nu[\frac{\partial^2 f}{\partial\xi^2} + \frac{\partial f}{\partial\xi}\int_{-\infty}^{\xi}[\frac{\partial f}{\partial\xi}]^2 = \sigma\int_{-\infty}^{\xi}\frac{\partial^2 f}{\partial\zeta^2} \qquad (5.57)$$

where f is a function of ξ, ζ and τ.

Differentiating Eq.(5.57) with respect to ξ we obtain,

$$[\frac{\partial f}{\partial\tau} + \frac{3}{2}\mu f^2\frac{\partial f}{\partial\xi} + \mu\frac{\partial^3 f}{\partial\xi^3} + \nu[\frac{\partial^2 f}{\partial\xi^2} + \frac{\partial f}{\partial\xi}\int_{-\infty}^{\xi}[\frac{\partial f}{\partial\xi}]^2]_\xi = \sigma\frac{\partial^2 f}{\partial\zeta^2} \qquad (5.58)$$

This equation is an integro-differential equation. In the absence of the ν term this equation reduces to the modified Kadomtsev-Petviashvili equation [66]. The ν term represents the effect of dissipation. It should be noted that this additional term is not only merely a higher derivative of f with respect to ξ but contains nonlocal nonlinear terms.

5.4 Steady state solutions

We first examine the existence of steady state solutions of this equation, we assume that $f(\xi, \zeta, \tau)$is a function of η through $(\eta - \lambda\tau)$, where λ is a constant. That is we seek solutions of Eq.(5.58) in a moving frame of reference. Under this assumption, Eq.(5.58) can be integrated once with respect to η subject to the boundary conditions f$\to$0 as $\eta \to -\infty$, giving rise to

$$-\lambda f + \frac{1}{2}\mu f^3 + \mu\frac{\partial^2 f}{\partial\eta^2} + \nu\frac{\partial^2 f}{\partial\eta^2} + \nu[\frac{\partial f}{\partial\eta} + f\int_{\infty}^{\eta} f^2] = \int\int\sigma\frac{\partial f}{\partial\eta}(d\eta)^2 \qquad (5.59)$$

We can now show that in the special case of $-\mu << \nu$ that is, when the effect of dispersion can be neglected compared to that of dissipation, Eq.(5.59) admits solutions of the form:

$$f(\eta,\tau) = \frac{-\lambda}{\nu} Sech^2 \sqrt{\frac{-\lambda}{\nu}}(\eta - \lambda\tau) \tag{5.60}$$

provided $\lambda < 0$ and there is no bounded solution if $\lambda > 0$.

Integrating Eq.(5.60), we have the following shock solution for θ

$$\theta = \frac{\pi}{2} - \sin^{-1}\left\{\tanh(\frac{\lambda}{\nu}\eta)\right\} \tag{5.61}$$

For the case of ν =0 we can arrive at the modified Kadomtsev -Petviashvili equation as

$$[\frac{\partial f}{\partial \tau} + \frac{3}{2}\mu f^2 \frac{\partial f}{\partial \xi} + \mu \frac{\partial^3 f}{\partial \xi^3}]_\xi = \sigma \frac{\partial^2 f}{\partial \zeta^2} \tag{5.62}$$

Eq.(5.62) is similar to the mKP equation obtained in chapter **2**. So the steady state solution of Eq.(5.61) can be obtained by using elliptic integrals. Thus we find:

$$f = 2k^2 sech^2 2k\eta \tag{5.63}$$

where $\eta = \xi + \zeta - \lambda\tau$, and $k^2 = \frac{\lambda - \sigma}{\mu}$, λ is a constant. Since $f = \frac{\partial\theta}{\partial\eta}$, θ is obtained as $\theta = \int_{-\infty}^{\eta} f d\eta$. θ increases from 0 to 2π or decreases from 0 to -2π according as $k > 0$ or $k < 0$ as η goes from $-\infty$ to $+\infty$

Also we can obtain explicit form of solutions for M_{0y}, M_{0z}, H_{0y}, H_{0z}, H_{1x} and M_{1x}. from Eq.(5.51-5.52). Thus we can obtain:

$$M_{0y} = m_0 sin\phi \cos(2k^2\sqrt{[1 - sech^2 k\eta]})$$

$$M_{0z} = m_0 sin\phi \sin(2k^2\sqrt{[1 - sech^2 k\eta]})$$

$$H_{0y} = \alpha m_0 \sin\phi \cos(2k^2\sqrt{[1 - sech^2 k\eta)]})$$

$$H_{0z} = \alpha m_0 \sin\phi \sin(2k^2\sqrt{[1 - (sech^2 k\eta)]})$$

$$H_{1x} = \frac{2Vk^2 sech^2 k\eta}{\mu_0 \gamma(1+\alpha)}$$

$$M_{1x} = -\frac{2Vk^2 sech^2 k\eta}{\mu_0 \gamma(1+\alpha)}$$

We have found that M_{1x}, H_{1x}, M_{0y} and H_{0y} components give soliton solutions while M_{0z} and H_{0z} components give kink solutions.

Energy of the solitary wave:

Any localized static (time -independent) solution is a soliatry wave [3]. All these solutions have localized energy density and finite total energy.

The energy expression for the mKP solitary wave is obtained by putting

$$f = U_\xi \tag{5.64}$$

in Eq.(5.61). Therefore from Eq.(5.62) we can write

$$E = \int_{-\infty}^{\infty} [\frac{1}{2}\mu(U_{\xi\xi})^2 - \frac{3}{24}\mu(U_\xi)^4 - \frac{1}{2}\sigma(U_\zeta)^2 d\eta \tag{5.65}$$

$$\begin{aligned} E = \tfrac{1}{6720}\ & k(\sec h^7[k\ x]\ [-1155\ \sin h[k\ x]+ \\ & 45360\ k^4\ \sin h[k\ x] - 1029\ \sin h[3\ k\ x]- \\ & 21168\ k^4\ \sin h[3\ k\ x] - 413\ \sin h[5\ k\ x]+ \\ & 3024\ k^4\ \sin h[5\ k\ x] - 59\ \sin h[7\ k\ x]+ \\ & 432\ k^4\ \sin h[7\ k\ x]]) \end{aligned} \tag{5.66}$$

The momentum expression for this solitary wave is given by

$$P = \int_{-\infty}^{\infty} [\frac{1}{2}\mu(U_\xi)^3 + \sigma U_\zeta] d\eta \tag{5.67}$$

That is,

$$P = \frac{1}{40}\ \sec h^5[x]\ (70\ k^2 \sin h[x] + 65\ k^2 \sin h[3\ x] + 19\ k\ \sin h[5\ x]) \tag{5.68}$$

5.5 Conclusion

We have studied the dynamics of the magnetization of a ferromagnet when an electromagnetic wave propagates through it in the presence of dissipation in (2+1) dimensions. A perturbation analysis is carried out to understand the nature of excitations due to the interaction between the magnetic field H and the magnetization M of the ferromagnet. We have analyzed the resulting equations separately for the dissipation dominant case and for the dispersion dominant case and obtained solutions in both cases. In the dissipation dominant case, the system reduces to that of Eq.(5.58) and gives shock wave solutions.

The resulting equation in dispersion dominant case is the modified Kadomtsev-Petviashvili equation. The mKP equation is found to be non-integrable in Painleve sense [24], but we can find the solitary wave solutions. Notice that the function f, which obeys the mKP equation can be considered as the derivative of the angle θ (the angle of procession of the whole magnetization density vector around the propagation direction) and also can be considered as the amplitude of the first order term M_1^x of the longitudinal component of the magnetization density. We have found that M_{1x}, H_{1x}, M_{0y} and H_{0y} components give soliton solutions while M_{0z} and H_{0z} components give kink solutions.

Chapter 6

Summary and conclusions

It is a capital mistake to theorize before one has data.

Sir Arthur Conan Doyle,

'The Adventures of Sherlock Holmes'

Much work on the electromagnetic waves propagating through various nonlinear media has been made so far. In particular, applying the reductive perturbation method established by Taniuti and Wei to the case of the electromagnetic wave propagation through a cold collisionless plasma in the presence of an external magnetic field is given much importance in this thesis. We have also analysed the case of electromagnetic wave propagation through ferromagnetic media.

In this thesis we have not considered the well studied models of plasma like the magnetohydrodynamic description or the electrostatic description of plasma. We have modelled plasma as an electron fluid. Using a new nonlinear perturbation expansion for the dependent variables, the system of equations describing the propagation of electromagnetic waves through a cold collisionless plasma has been reduced to the well known modified Korteweg-deVries equation at the lowest order of perturbation. The system can sustain two types of modes of propagation. We have analysed long wavelength modes with finite velocity In one case, the system of equations can be reduced to mKdV equation, but for the other case, the nonlinear evolution equation is given by the KdV equation. Both the mKdV and KdV equations describe the asymptotic behavior of long waves in the presence of nonlinear and dispersive effects and they have soliton solutions. The above calculation have been extended to (2+1) dimensions. In (2+1) dimensions the resulting equations are the mKP and KP equations respectively. The mKP equation is found to be not satisfying the Painleve test of integrability. But we have obtained the solitary wave solution of this equation. The amplitude of the field components shows both solitary wave as well as kink type excitations. The stability analysis of the solitary wave solution of this mKP equation under an external perturbation have been carried out in the third chapter of the thesis. The instability criterion have been analysed and is dependent on the system parameters. The growth rate of instability, i.e, for what values of Ω_1 the solitary wave become unstable, have been found. The graph in chapter **3** shows the behaviour of the solitary wave under external perturbation.

The interaction between the solitary wave solutions of the mKdV and the KdV equations have been investigated. Eventhough the interaction is taking place between two solitary wave solutions of two nonlinear evolution equations with different velocities of the same system, but we can not expect the usual soliton-soliton interaction behaviour in this scenario. Here one of the solitary wave, represented by the mKdV equation, will remain as a solitary wave after interaction, while the other wave after interaction will be represented by a long wave pulse containing a very large number of solitons, and the present treatment of the interaction system disregard the solitonic nature of this pulse, and can act only on an averaged amplitude of the wave.

In the case of electromagnetic wave propagating through ferrites, we have studied the dynamics of the system when an external perturbation is applied in (2+1) dimensions. The resulting equation in this case is an integro-differential equation. In the dissipation dominant case we have found that the magnetic fluctuations take place in the form of shock waves. But in the case of dispersion dominant case the equation is reduced to the mKP equation and possesses solitary wave solutions. The energy and the momentum of the solitary wave solution of the mKP equation are also found out.

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