# Flexure-shear Analysis of Concrete Beam Reinforced with GFRP bars

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ABSTRACT: This paper gives the details of flexure-shear analysis of concrete beams reinforced with GFRP rebars. The influence of vertical reinforcement ratio, longitudinal reinforcement ratio and compressive strength of concrete on shear strength of GFRP reinforced concrete beam is studied. The critical value of shear span to depth ratio (a/d) at which the mode of failure changes from flexure to shear is studied. The failure load of the beam is predicted for various values of a/d ratio. The prediction show that the longitudinally FRP reinforced concrete beams having no stirrups fail in shear for a/d ratio less than 9.0. It is expected that the predicted data is useful for structural engineers to design the FRP reinforced concrete members.

#### 1 INTRODUCTION

The fiber reinforced plastic (FRP) is a high strength, light weight material transparent to magnetic fields and radio frequencies. Due to its non-corrosive nature, the use of FRP bars in concrete is becoming popular when the structure is exposed to deicing or marine salts and other chemicals. As the modulus of elasticity of FRP bars are lower than the conventional steel bars, the structures reinforced with FRP bars show increased deflection when compared to similar structures reinforced with steel bars. The usage of FRP bars is limited due to the fact that it is to be cast in required shape at the manufacturing plant itself and cannot be bent at the work site (AslanFRP 2010). To account for the possible catastrophic failure in the FRP bars, the most FRP reinforced structures are recommended to design as over-reinforced sections. The design of FRP reinforced structures is often controlled by serviceability limits on deflection and crack width. Due to the superior corrosive resistance of FRP bars, the permissible crack widths in beams reinforced with FRP bars are higher when compared to that in steel reinforced concrete beams (Bank 2006). The recommendations for the design of FRP reinforced concrete structure are given in ACI 440.1R (2006). The codes of practice for the design of steel reinforced concrete structures are ACI318 (2008) and IS456 (2000).

El-Sayed, El-Salakawy and Benmokrane (2005) observed that shear strength of the concrete slabs reinforced with FRP bars increases with the increase in the amount of longitudinal reinforcements. Wegian and Abdalla (2005) found that the neutral axis

depth of GFRP reinforced concrete beams is very El-Sayed, El-Salakawy and Benmokrane (2006) showed that the shear strength of concrete beams increases with the increase in reinforcement ratio. Nehdi, Chabib and Said (2007) stated that the code of practice (ACI 440.1R 2006) estimates the conservative results for the computation of shear strength of FRP reinforced concrete beam. Hoult, Sherwood, Bentz and Collins (2008) indicated that a strong correlation exists between strain effect and the shear capacity of the beam. Tavares, Giogno and Paultre (2008) found that the strength of GFRP reinforced concrete beams is lesser than the strength of corresponding steel reinforced concrete beams. Ahamed, El-Salakawy and Benmokrane (2010) found that the shear strength of the beams is greater in beams with lesser spacing of GFRP stirrups. The flexure shear analysis of the FRP reinforced concrete beams is limited. This study addresses this gap in the literature.

## 2 ANALYTICAL MODEL

The details of the analytical model proposed in ACI440.1R(2006) are given.

## 2.1 Shear strength

The nominal shear strength  $(V_n)$  of the concrete section is computed by

$$V_{n} = V_{c} + V_{f} \tag{1}$$

where  $V_c$  is the shear strength of concrete section without stirrups and  $V_f$  is the shear resistance offered by GFRP stirrups. The  $V_c$  is computed by

$$V_c = 5(f_c')^{\frac{1}{2}} b c \le 8 (f_c')^{\frac{1}{2}} b d$$
 (psi & in) (2)  
$$V_c = 0.4 (f_c')^{\frac{1}{2}} b c \le 0.66 (f_c')^{\frac{1}{2}} b d$$
 (MPa & mm)

where  $f_c$ ' is the cylinder compressive strength, b is the width of the beam, c is the depth of neutral axis in the cracked elastic section and d is the effective depth of the beam.  $f_c$ ' is computed by

$$f_{c}' = 0.8 f_{ck}$$
 (3)

where  $f_{ck}$  is the characteristic compressive strength of concrete based on cube specimens. c is computed by

$$c = kd (4)$$

$$k = [(\rho_f \eta_f)^2 + 2(\rho_f \eta_f)]^{1/2} - (\rho_f \eta_f)$$
 (5)

where k is the neutral axis depth ratio,  $\rho_f$  is the FRP longitudinal reinforcement ratio and  $\eta_f$  is the modular ratio

$$\rho_{f} = A_f / bd \tag{6}$$

$$\eta_{\rm f} = E_{\rm f} / E_{\rm c} \tag{7}$$

where  $E_f$  is the longitudinal modulus of FRP bar and  $E_c$  is the modulus of elasticity of concrete.  $E_c$  is given by

$$E_c = 57000 (f_c')^{\frac{1}{2}}$$
 (psi) (8)  
 $E_c = 4700 (f_c')^{\frac{1}{2}}$  (MPa)

 $V_{\rm f}$  is computed based on 45 degree crack angle and is given by

$$V_f = \rho_v f_{fv} bd \tag{9}$$

$$\rho_{v} = A_{fv} / (bs_{v}) \tag{10}$$

where  $\rho_v$  is the vertical reinforcement ratio,  $A_{fv}$  is the area of the vertical stirrups across the shear crack,  $f_{fv}$  is the stress in the FRP stirrups and  $s_v$  is the spacing of vertical stirrups measured along the beam axis.  $f_{fv}$  is computed by

$$f_{fv} = MIN [0.004 E_f, f_{fb}]$$
 (11)

where  $f_{\text{fb}}$  is the tensile strength of FRP rebar with a bend and is given by

$$f_{fb} = f_{fu} \left[ 0.05(r_b/_{db}) + 0.3 \right]$$
 (12)

where  $f_{fu}$  is the design tensile strength of FRP bar,  $r_b$  is the inside radius of the bend of the stirrup and  $d_b$  is the diameter of the FRP rebar of the stirrup. In Eq. (12), the magnitude of  $(r_b/d_b)$  shall not be less than 3.0.  $f_{fu}$  is given by

$$f_{fu} = C_E f_{fu} * \tag{13}$$

where  $C_E$  is the environmental reduction factor and  $f_{fu}^*$  is the guaranteed tensile strength of FRP bar. The value of  $C_E$  depends on the exposure conditions and the type of fibers and is given in ACI440.1R (2006).

# 2.2 Flexure strength

The nominal shear resistance  $(V_n^*)$  corresponding to the flexural capacity of a reinforced concrete beam subjected to concentrated load is given by

$$V_n^* = M_n / a \tag{14}$$

where  $M_n$  is the nominal moment of resistance of the section and a is the shear span.  $M_n$  is computed by

$$M_n = A_f f_f (d-a^*/2)$$
 (15)

where  $A_f$  is the area of the longitudinal FRP bars,  $f_f$  is the stress in the longitudinal bar and  $a^*$  is the depth of Whitney's stress block.  $f_f$  is given by

$$f_f = [(E_f \varepsilon_{cu})^2/4 + 0.85 \beta_1 f_c' (E_f \varepsilon_{cu})/\rho_f]^{1/2} - 0.5(E_f \varepsilon_{cu})(16)$$

where  $\varepsilon_{cu}$  is the ultimate strain in concrete and is equal to 0.003.  $\beta_1$  is a factor depending on the concrete strength ( $f_c$ ') and is given by

$$\beta_1 = 0.85$$
 when  $f_c' < 4000 \text{ psi}$  (17)

 $\beta_1 = 0.85 - 0.05 \; (f_c\text{'-}4000)/1000 {\ge} 0.65 \qquad \text{when } f_c\text{'} {\ge} 4000 \; psi$ 

$$\beta_1 = 0.85$$
 when  $f_c' < 28$  MPa

$$\beta_1 = 0.85 - 0.05 (f_c'-28)/7 \ge 0.65$$
 when  $f_c' \ge 28$  MPa

In Eq. (15), a\* is given by

$$a^* = A_f f_f / (0.85 f_c' b)$$
 (18)

#### 2.3 Failure load

The failure load of the beam (P) subjected to four point bending as shown in Fig. 1 is computed by

$$P = 2 * MIN (Vn, Vn*)$$
 (19)

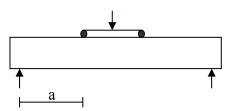


Figure 1. Schematic of loading in RC beam

The influence of various parameters on the failure load predicted based on the model proposed by ACI440.1R (2006) is analysed.

## 3 DETAILS OF BEAM

A concrete beam of size 600mm x 220mm having an effective depth of 500mm is used. The cross sectional details of the beam are given in Fig 2. The guaranteed tensile strength of GFRP bars in the transverse direction is taken as 760 MPa and strength of bar in the longitudinal direction is 655 MPa (AslanFRP 2010).

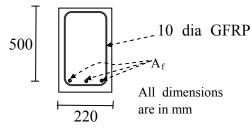


Figure 2. Cross sectional details of the RC beam

The variables considered for the study are longitudinal reinforcement ratio  $(\rho_f)$ , vertical reinforcement ratio  $(\rho_v)$ , and concrete compressive strength  $(f_{ck})$  and shear span to depth ratio (a/d). GFRP longitudinal reinforcement ratio  $(\rho_f)$  is varied between 1 and 4 percent. The vertical GFRP reinforcement ratio  $(\rho_v)$  of 0 to 1 percent is considered. The grade of the concrete  $(f_{ck})$  is varied between 25 and 75 MPa. Failure load of the reinforced concrete beams with a/d ratio varying between 1.0 and 12.0 is predicted.

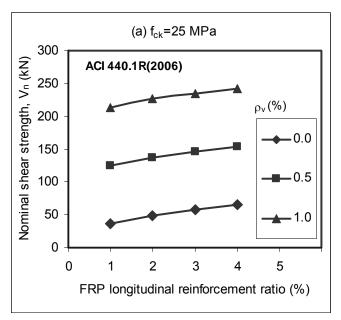
# 4 RESULTS AND DISCUSSIONS

The nominal shear strength and failure load of the beam of size 220mm x 600mm has been computed using the model proposed in ACI 440.1R (2006). The influence of FRP reinforcement ratio on the shear strength of normal strength concrete beam has been evaluated ( $f_{ck} = 25$  MPa) and is given in Fig 3(a). The shear strength of the beam with GFRP reinforcement increases with the increase in the vertical and longitudinal reinforcement ratio. The influence of longitudinal reinforcement on shear strength of FRP reinforced concrete beam is nominal.

The influence of concrete strength ( $f_{ck}$ ) on the shear strength of beam reinforced with GFRP bars is given in Fig 3(b). The magnitude of vertical reinforcement ratio is taken as 0.5 percent for the prediction of shear strength of beam given in Fig 3(b). The influence of concrete strength on the shear strength of FRP reinforced beam is limited.

The flexure shear analysis of the FRP reinforced beam is carried out and the results are given in Fig 4. The failure load (P) of the beam is predicted corresponding to the different values of shear span to depth ratio (a/d). The horizontal plateau of the graph shown in Fig 4 indicates the shear failure and the sloped portion indicates the flexure failure in the beam. The starting point of sloped regime represents the a/d ratio at which the mode of failure changes from shear to flexure. The influence of the FRP longitudinal steel reinforcement ratio is given in Fig 4(a). The failure load of the beam increases with the increase in FRP longitudinal reinforcement ratio. The a/d ratio at which the mode of failure changes from shear to flexure increases with the increase in FRP longitudinal reinforcement ratio.

The influence of concrete strength  $(f_{ck})$  on failure load of the beam (P) is given in Fig 4(b). The failure load of the beam increases with the increase in the concrete strength. The a/d ratio at which the mode of failure changes from shear to flexure increases with the increase in concrete strength.



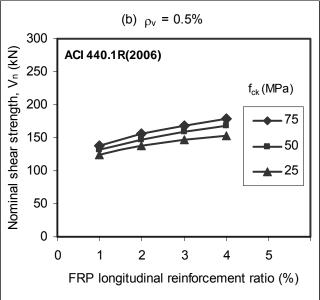
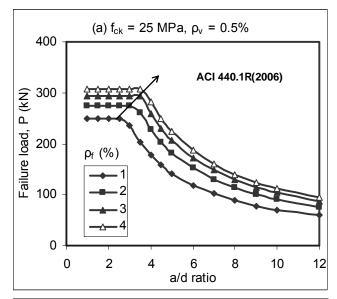
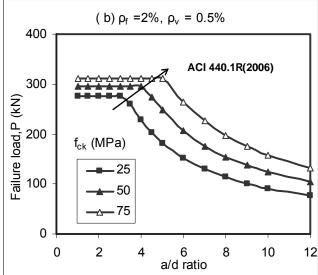


Figure 3. Predicted nominal shear strength of GFRP reinforced concrete beam

The variation of the failure load of the beam (P) corresponding to various values of vertical reinforcement ratio  $(\rho_v)$  is given in Fig 4(c). For initial values of a/d ratio, the failure load in beams having 1 percent vertical reinforcement is found to be greater in magnitude when compared to the corresponding beam having 0.5 percent vertical rein-The a/d ratio corresponding to the change in mode of failure from shear to flexure depends on the amount of vertical reinforcement. The predicted failure load of the beam corresponding to the higher values of a/d ratio is found to be equal in magnitude for all the values of vertical reinforcement ratio  $(\rho_v)$  and is represented by a single line in the inclined regime. This is expected because the flexural failure load depends on the amount of longitudinal reinforcement and not on the vertical reinforcement.





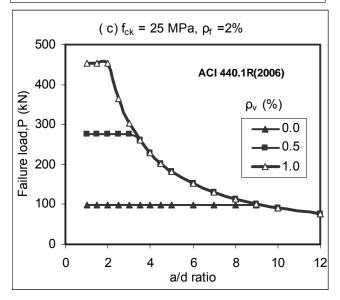


Figure 4. Predicted failure load of GFRP reinforced concrete beam

#### 5 CONCLUSIONS

Based on the present study the following conclusions are arrived at.

- Effect of FRP reinforcement in the transverse direction on the shear strength of reinforced concrete beam is significant.
- Influence of FRP reinforcement in the longitudinal direction on the shear strength of FRP reinforced beam is nominal.
- The increase in shear strength of beam with the increase in concrete strength is limited.
- The a/d ratio corresponding to the change in mode of failure increases with the increase in the longitudinal or transverse FRP reinforcement.
- The prediction based on the model proposed by ACI 440.1R (2006) indicates that the FRP reinforced concrete beams having no transverse reinforcement fails in shear if the a/d ratio is less than 9.0.

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